Electronic technology

principal contents

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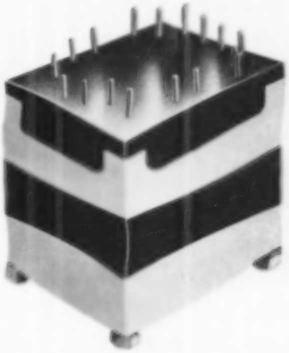

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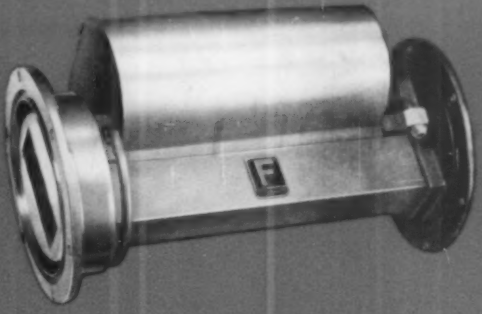

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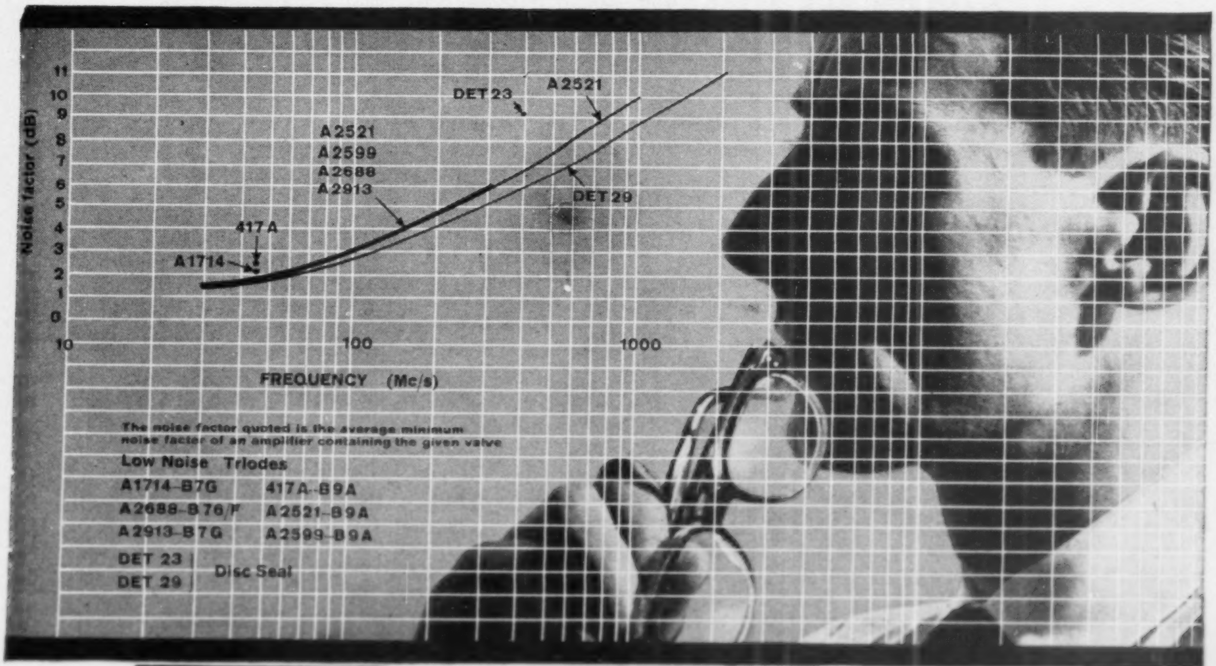

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range	*(nominal)*	*30% down*	*50% down*	*Sensitivity*
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15 V	×10	5 Mc/s	7 Mc/s	3 V/cm (3·33 mm/V)
5 V	×30	5 Mc/s	7 Mc/s	1 V/cm (1 cm/V)
1·5 V	×100	5 Mc/s	7 Mc/s	0·3 V/cm (3·33 cm/V)
500 mV	×300	5 Mc/s	7 Mc/s	0·1 V/cm (10 cm/V)
150 mV	×1000	5 Mc/s	7 Mc/s	30 mV/cm (33·3 cm/V)
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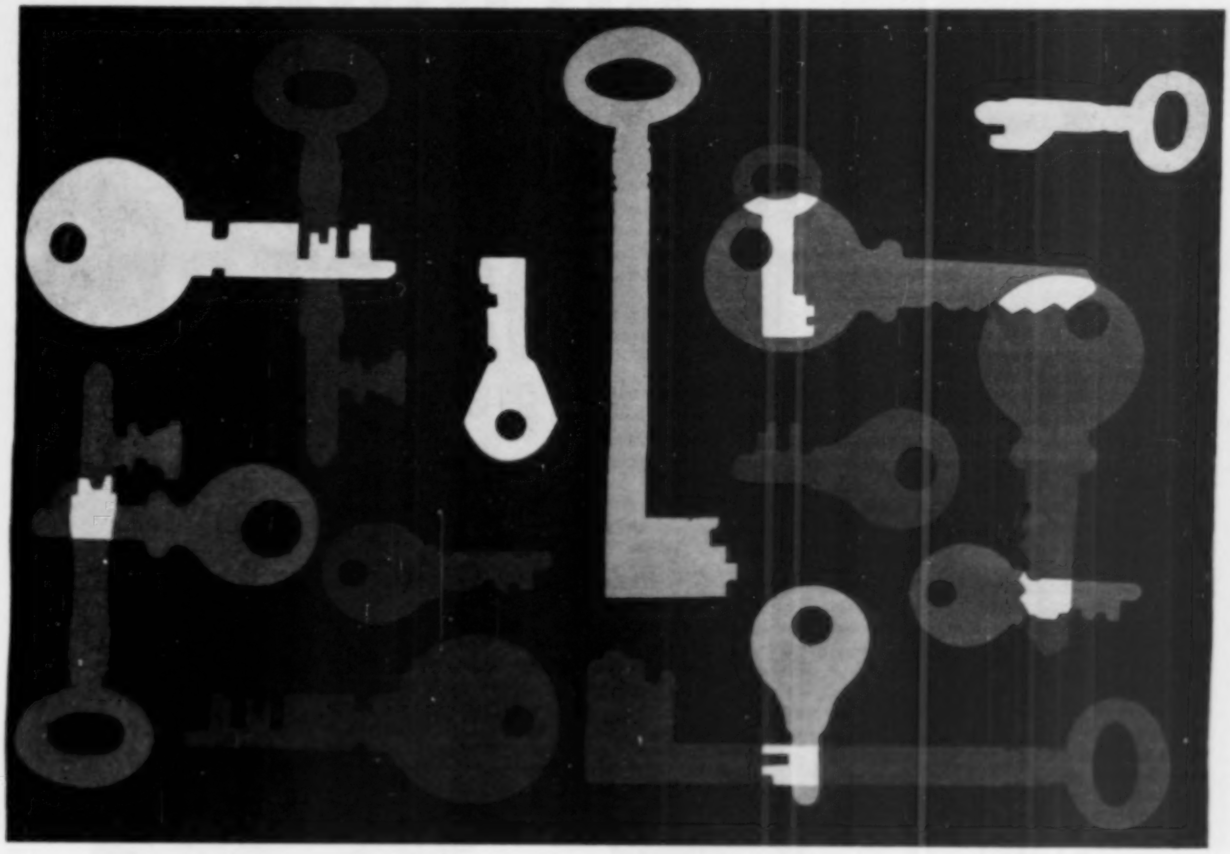

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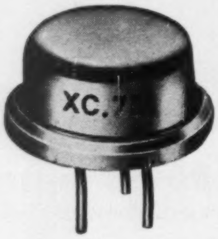

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Established 1923

Published on the fifth of each month

ANNUAL SUBSCRIPTION
(including Annual Index to Abstracts and References)
Home and Overseas £3 7s 0d, Canada and U.S.A. $9.50
Second-class mail privileges authorized
at New York, N.Y.

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Published by
ILIFFE ELECTRICAL PUBLICATIONS LTD. *Managing Director:* H. S. Pocock M.I.E.E., Dorset House, Stamford Street, London S.E.1
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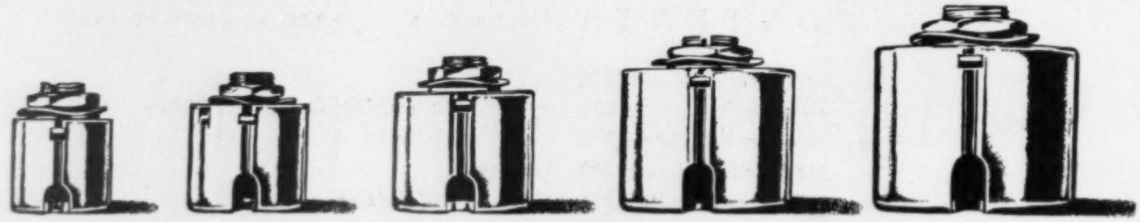

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Satellite Communications

THE use of artificial satellites as relay stations in point-to-point communications is now receiving a great deal of attention—and publicity. It formed a large part of the recent Convention organized by the British Institution of Radio Engineers.

It appears hardly practicable to use a satellite as a passive reflector and most systems envisage that it will carry a receiver and transmitter. In view of the high cost of launching a satellite, the equipment must not only be extremely compact but very reliable. One would hardly contemplate such a scheme without being fairly sure that it will work for at least five years unattended.

Quite obviously, a major problem is the power supply. Chemical batteries appear rather impracticable and we are sure that most people's thoughts turn to solar cells. Although their efficiency is only around 11%, they are very practicable save for one thing; they are very liable to damage by radiation from the van Allen belt. It is estimated that even when some protection is used the efficiency is halved after 10 months' operation.

An alternative which is being tried is a nuclear energy source of power. Perhaps this will be the answer to the problem, but it is early days yet.

Because of this difficulty of power supply it is necessary to keep the power requirements at a minimum. A major factor here is the proper choice of the operating frequency. Too low a frequency means excessive galactic noise and absorption in the ionosphere. Too high a frequency means excessive atmospheric absorption. There are very many factors involved but it looks as if the frequency ought to be in the range 1–6 kMc/s (30–5 cm) and international agreement on frequency allocation is needed.

It is not only solar cells which are damaged by radiation, of course. They are the most vulnerable, for they are necessarily exposed on the exterior of the satellite. Other parts are screened in some degree, but apparently there is no part which can be said to be entirely unaffected by radiation. A good deal of research is going on into the problem and will, no doubt, result in improved components.

There is today, no doubt at all that it is possible to use a satellite as an active relay station. Whether it is yet, or will ever be, economical to do so is, in our opinion, still open to doubt.

Magnetic Recording

THEORY OF TAPE MAGNETIZATION

By L. G. Sebestyen, Dipl. Ing., A.M.I.E.E. and J. Takacs, A.M.I.E.E.

SUMMARY. *A novel explanation for the magnetization of the magnetic tape and the role of a.c. bias is suggested and an attempt is made to present the problem in a form suitable for mathematical analysis. In spite of gross simplifications, several hitherto unexplained phenomena can be accounted for on the basis of this analysis. A simple experiment proves the salient points of the theory.*

In 1927 Carlson and Carpenter[1] patented their invention whereby distortion is reduced, and the linearity and signal-to-noise ratio of magnetically-recorded signals is improved by impressing an alternating current 'bias' on to the recording medium in addition to the signal. A number of explanations of the mechanism of magnetic recording using high-frequency a.c. bias have been published in the past. Toomin and Wildfeuer[2] draw attention to the rise of minor hysteresis loops as the result of the superimposed low-frequency and high-frequency fields. Similar approach to the problem was made by Wetzel[3], Camras[4,5] and more recent contributions are those of Oerding[6] and Greiner[7]. Holmes and Clark[8] offer an analogy with the push-pull operation of valves. Axon's[9] detailed analytical account draws attention to the fact that the final magnetization of a particle depends on its magnetic history from the time it enters the record-head gap and that a number of cycles are required to produce steady-state conditions as regards the hysteresis loop in any tape element after having been subjected to the initial magnetizing force on entering the magnetic field. Daniel[10] describes a method of calculating the surface induction corresponding to any value of wavelength, bias current, oxide thickness and recording gap width, taking into account the distortion of the recording field within the recording medium, and the wavelength-dependent demagnetization losses.

Westmijze[11,12] states that the recording takes place at a point within or beyond the recording gap where the biasing field amplitude is equal to the value of H_{crit}, which makes the remanence proportional to the signal, where H_{crit} is the field-strength below which no remanent magnetization is retained in the recording medium.

A generalization and mathematical analysis of Westmijze's theory is given by Schwantke[13] who proves analytically that when a.c. bias is applied, the remanent magnetism is proportional to the signal field-strength, even if the hysteresis loop is not approximated by straight and parallel lines, as in Westmijze's articles. He concludes that the B-H loop is not suitable for description of the recording process and a different approach to the problem is made in the second part of his paper, based on Preisach's model of ferromagnetic substances. Preisach's model attributes a rectangular hysteresis loop to each Weiss-domain with H_b coercive force and H_m asymmetry with respect to the zero ordinate. The recording process is followed in the H_m-H_b system of co-ordinates assuming that each domain has 'positive' or 'negative' orientation, depending on the instantaneous value of field strength. Woodward and Della Torre[14] prove that the non-symmetrical hysteresis loops are the consequence of local fields of neighbouring particles. Finally, in a recent paper Bedford[15] gives a simple and new physical picture on the role of a.c. bias, but no mathematical approach is made. The theories listed above are, to a certain extent, contradictory; there are gaps in understanding of the recording process and a unified treatment of the problem which gives satisfactory explanation to a number of experimental facts is still missing. An attempt is made

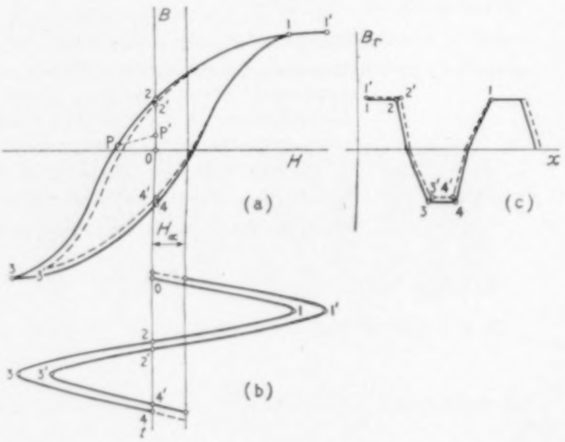

Fig. 1. *The recording process: (a) is the hysteresis loop of the magnetic medium; (b) the bias and the bias + d.c. signal waveform, and (c) the corresponding magnetization retained by the medium*

here to introduce a novel explanation and an analytical treatment of the problem. Bedford's observations are used as a starting point.

A Novel Explanation of the Recording Process

Let us first summarize the salient point of the novel theory, *which is that fundamentally it is the bias waveform* which is written on the tape while the signal appears as 'modulation' of this carrier. The replay channel, especially the replay head, acts as a low-pass filter and extracts the information from the modulated carrier.

During the recording process when the bias is already switched on but no information signal is present, any element of the tape finds itself cycled round the hysteresis loop a certain number of times during its passage in front of the recording gap until it arrives at the trailing edge where the field is supposed to collapse abruptly to zero, (Fig. 1).

Fig. 1 (a) represents an approximate hysteresis loop of a commercial-grade magnetic tape of medium coercivity. Any element which leaves the trailing edge during the quarter-cycle 1-2 retains the magnetization corresponding to 2. The remanent magnetization of the element corresponding to 2-3 which leaves during the next quarter-cycle is rather uncertain. Point P' is the estimated value to which P collapses. During the next quarter-cycle 3-4 remanent magnetization corresponding to point 4 is again retained and finally the last quarter-cycle repeats 2-3 in mirror image.

Thus applying a purely sinusoidal bias, sufficient to saturate the tape, will result in a remanent magnetization on the tape which can be regarded as approximately a trapezoidal waveform. Now if a signal of low frequency is added to the bias (see dotted lines in Fig. 1) the consequence is a small distortion of the previously symmetric magnetization waveform on the tape; i.e., the regions of constant magnetization are now of unequal length and the slope of the trapezoidal waveform becomes slightly altered. If the bias is not sufficient to saturate the tape, the amplitudes of the two halves are unequal. All of these effects produce a d.c. component in the waveform which is recorded on the tape.

This over-simplified picture of the recording process entirely neglects such effects as the self-demagnetization of the recorded signal and does not take into account that property of the medium, wherein its magnetization will not follow a sharp rise in the magnetic field. It further neglects the field distribution in and around the gap of the recording head. In spite of this, it can be considered as a first approximation of the magnetization of the medium, as will be verified later.

LIST OF SYMBOLS

B	Magnetic induction
B_r	Remanent magnetic induction
B_y	Surface induction
H	Magnetic field-strength
H_c	Coercive force
H_s	Saturation field-strength
T, t, τ	Time
τ_0	Amplitude of signal frequency
ω	Angular frequency of the signal
ν	Angular frequency of the bias
J_0, J_1, J_2	Bessel functions
ϕ_r	Remanent flux
a_0, a_n, b_n	Coefficients of the Fourier series
v	Speed of the recording medium
c, c_1	Proportionality constants
ϵ	Replay output voltage
α	Distance along the path of the recording medium

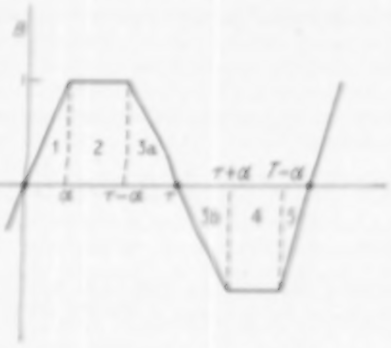

Fig. 2. Notations used in the mathematical analysis

Mathematical Analysis

The trapezoidal wave of Fig. 1 can be expressed as a series of the form

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{t} t \right) \quad (1)$$

Using the notations of Fig. 2 this becomes:

$$a_0 = \frac{2}{T} (2\tau - T) \quad \ldots \quad (2)$$

$$a_n = \frac{T}{\pi^2 n^2 \alpha} \sin \frac{2\pi n}{T} \tau \sin \frac{2\pi n}{T} \alpha \quad \ldots \quad (3)$$

$$b_n = \frac{T}{\pi^2 n^2 \alpha} \left(\sin \frac{2\pi n}{T} \alpha - \cos \frac{2\pi n}{T} \tau \sin \frac{2\pi n}{T} \alpha \right) \quad (4)$$

Assuming now $\tau = \frac{T}{2} + \tau_0 \sin \omega t$ signal frequency,

$$a_0 = \frac{4}{T} \tau_0 \sin \omega t \quad \ldots \quad (5)$$

$$a_n = \frac{2}{n\pi} \frac{\sin n\nu\alpha}{n\nu\alpha} (-1)^n [2 J_1(K) \sin \omega t + 2 J_3(K) \sin 3 \omega t + \ldots + J_{2m+1}(K) \sin (2m+1) \omega t] \quad \ldots \quad (6)$$

$$b_n = \frac{2}{n\pi} \frac{\sin n\nu\alpha}{n\nu\alpha} \left\{ 1 - \left[(-1)^n [J_0(K) + 2 J_2(K) \cos 2 \omega t + \ldots + 2 J_{2m}(K) \cos 2 m \omega t] \right] \right\} \quad \ldots \quad (7)$$

Here $J_0(K)$ $J_1(K)$, etc., are Bessel-functions of the first kind and

$$K = \frac{2\pi n}{T} \tau_0 = n \nu \tau_0$$

Now if $A = \frac{2}{n\pi} \frac{\sin n\nu\alpha}{n\nu\alpha}$

then the waveform on the tape may be expressed as

$$f(t) = \frac{\nu}{\pi}\tau_0 \sin \omega t +$$

$$\sum_{n=1}^{\infty} A\left\{(-1)^n \sum_{m=1,3,5\ldots}^{\infty} \mathrm{J}_m(K)\,[\sin(m\omega+n\nu)t + \sin(m\omega-n\nu)t]\right.$$

$$+ \sin \nu nt - (-1)^n \sum_{m=0\,2,4\ldots}^{\infty} \mathrm{J}_m(K)\,[\sin(m\omega+n\nu)t$$

$$\left. + \sin(m\,\omega - n\,\nu)t]\right\} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

It must not be forgotten that both A and K which were introduced for the sake of brevity are functions of n.

As Equ. (8) represents the remanent flux density in the tape, then the surface density B_y becomes, by differentiation of Equ. (8)

$$B_y = \frac{d\phi_r}{dx} = \frac{\partial \phi_r}{\partial t}\frac{\partial t}{\partial x} = c.\frac{\omega}{v}\frac{\nu}{\pi}\tau_0 \cos \omega t$$

$$+ \frac{c}{v}\sum_{n=1}^{\infty} A\left\{(-1)^n \sum_{m=1,3,5\ldots}^{\infty} \mathrm{J}_m(K)\,[(m\,\omega+n\nu)\cos(m\,\omega+n\nu)t\right.$$

$$+ (m\,\omega-n\nu)\cos(m\,\omega-n\nu)t] + \nu n \cos \nu n\, t$$

$$- (-1)^n \sum_{m=0,2,4}^{\infty} \mathrm{J}_m(K)\,[(m\,\omega+n\nu)\cos(m\,\omega+n\nu)t$$

$$\left. + (m\,\omega - n\nu)\cos(m\,\omega - n\nu)t]\right\} \quad \ldots \quad \ldots \quad (9)$$

where c is a proportionality constant.

According to the above equation, the spectrum of the B_y waveform which is written on the tape (neglecting again the demagnetization effects) consists of the signal frequency itself, with the wide range of sidebands which are grouped around ν, the bias frequency and its multiples, and their amplitude are proportional to Bessel-functions of the first kind. The replay head and amplifier acts as a low-pass filter and eliminates all frequencies which are beyond $k\omega_{max}$ where ω_{max} is the maximum signal frequency and k is a constant, depending on the replay head and amplifier properties. If we substitute $n = 1$, the replay output will consist of components with frequency of ω, $\nu \pm \omega$, $\nu \pm 2\omega$, etc., with amplitudes corresponding to the relevant Bessel functions $\mathrm{J}_m(\nu\,\tau_0)$ multiplied by $(\sin\alpha)/\alpha$.

If the ratio of ν/ω is not chosen properly, the $\nu \pm \omega$, $\nu \pm 2\omega$, etc., components may be within the transmission band of the replay channel and may be the origin of the 'beat-notes'. (Another cause of the beat-notes is that in the record head combination frequencies of bias and signal and their harmonics will appear and these will be recorded on the tape.) The first term of the expression for B_y is of principal interest as this contains the recorded signal itself. Re-writing it and multiplying with the tape speed v, we obtain for the fundamental component of the replay output voltage the following expression

$$\epsilon = c_1\,\omega\,\frac{\tau_0}{T}\cos \omega t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Thus, assuming that the recording is a pure pulse-width modulation process, ignoring the pulse-amplitude and pulse-slope variation, and further neglecting all fringing effects, demagnetization and all other losses, we find that the replay output voltage is

(a) proportional to the signal amplitude

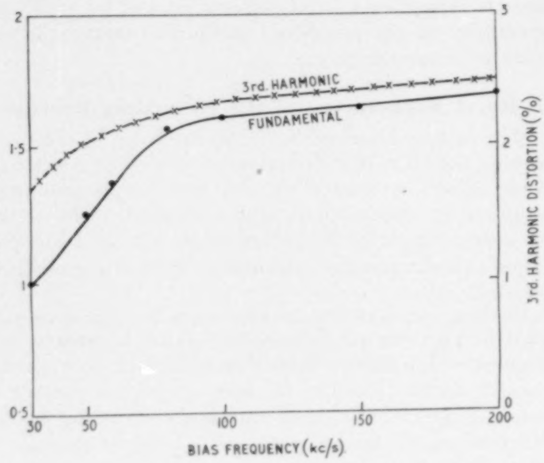

Fig. 3. *The measured increase of replay output voltage and 3rd-harmonic distortion against bias frequency. Bias current was adjusted to maximum replay output and signal input kept constant. Because of the low-loss ferrite record head, the bias current did not need more than 10% adjustment*

(b) increases at 6dB/octave rate with signal frequency

(c) increases at 6dB/octave rate with bias frequency

Points (a) and (b) are well-known facts. The novelty is in point (c), and the increase with replay output voltage with increasing bias frequency remains to be proved by the experiments. One would expect that—because of the approximations made during the analysis—this will apply for low bias frequencies only, and approach a limit asymptotically.

We are now in a position to draw some further conclusions from the physical picture of the magnetization process. The basic assumption is that the bias (the peak level of which is high enough to drive the recording medium into saturation) is recorded on the tape and as a consequence of the non-linearity of the B–H loop and near-saturation level, it appears as a trapezoid-like waveform of magnetization. An increase in the bias amplitude will cause the trapezoid waveform to approach a square wave and a decrease will make it like a triangular wave. If it is approaching a square wave, the asymmetry caused by the signal will be less and less (re-drawing Fig. 1 will prove this). On the other hand, the signal will not cause any modulation of the triangular wave. Between these two extremes there is an 'optimum' bias which allows maximum modulation; hence the sensitivity of the replay output voltage to the bias level. As to the distortion, the physical picture of the magnetization suggests that a higher bias level (longer 'flat' portions in Fig. 1) will tolerate a higher signal level, and an increase in bias frequency will increase the distortion (for constant input-signal level). The gap width of the recording head is of secondary importance.

Experimental

Only the simplest experiments have been carried out which needed no other tools but a conventional tape recorder.

The experiments and some conclusions were as follows:

1. A recording head (with low losses at high frequencies) was employed for recording a 1,000-c/s sinusoidal signal, using a continuously-variable bias source (record head gap: 0.0005 in., tape speed: 30 in./sec. '3M' type 159 tape). The bias current was adjusted for maximum replay output voltage.

2. In order to obtain some information on magnetization of the tape during the recording process, various signals were recorded on the tape, the replayed waveform observed on an oscilloscope, and the well-known technique of depositing fine iron powder on the tape, in order to make the recording visible, was employed.

The signals recorded on the tape were as follows:

(i) 960 c/s without bias, high level to saturate the tape.

(ii) 960 c/s with 5-kc/s, 10-kc/s and 50-kc/s bias, adjusted to optimum replay output (5% distortion level).

(iii) The signal was replayed through (1) a narrow band filter, centred around 960 c/s, and (2) a wide-band amplifier.

Experiment 1

The results of experiment 1 are shown in Fig. 3. As expected, the replay output increases with bias frequency but the rate of increase is less than 6dB/octave even at low bias frequencies. Quite obviously, the magnetization of the tape cannot follow very high frequencies, and the higher the bias, the less valid is the approximation of the magnetization waveform to a sharp-edged trapezoid. Further, the wavelength-dependent self-demagnetization is entirely neglected.

Experiment No. 1 was repeated with a different record head which had a gap of 0.0015 in. The results proved that the effect is not related to the record-head gap width. Further, no difference could be detected in the replay output signal, depending on the bias frequency as long as it was much higher than the signal frequency other than increase in gain and distortion with increasing bias frequency.

It seems that the gross effect of the a.c. bias on the recording mechanism is not influenced by the number of bias cycles to which an element of the magnetic tape is subject during its passage in front of the gap. This is in agreement with the observation that the sharpness of the trailing edge is of importance at the record head. If, as a first approximation, we assume that the bias field occupies uniformly the entire recording gap (the actual field distribution in the recording gap is well known, see for instance refs. 16 and 17) then more than 5 full bias cycles are completed during the passage of an element of the tape which is equal to the gap width when employing 300 kc/s and only a fraction of one period at 10 kc/s. (These two were the lowest and highest bias frequencies employed in the experiment).

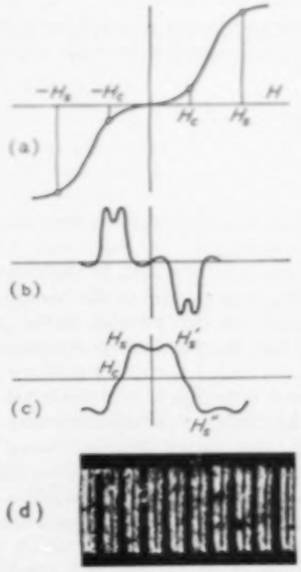

Fig. 4. *The remanence curve (a); the observed replay output voltage on oscilloscope screen for a saturated signal recorded without bias (b); the corresponding surface induction obtained by graphical integration from (b) (c); and the iron-powder pattern (d)*

Experiment 2

The results of Experiment No. 2 are shown in Fig. 4. The results need some interpretation.

Remembering the remanence curve of a conventional magnetic oxide [Fig. 4(a)] it is obvious that no permanent magnetization is retained below H_c. From H_c to H_s flux lines emanate from the tape and iron powder will be collected here. Beyond H_s the tape is saturated, the signal field reaches its peak and descends to H_s. In the middle region of the H_s–H_s' interval flux lines do not emanate from the tape and no iron powder will be attracted here. The same is repeated during the negative half-cycle of the signal. Therefore, closely-spaced double lines of iron powder will appear in the pattern. (The appearance of double lines is very characteristic for a saturated tape and is often used as a check for saturation. A further set of measurements proved that when reducing the signal level, the lines corresponding to H_s and H_s' appear more closely to each other and for a non-saturated signal only single lines will appear, at a distance corresponding to half-wavelength). The waveform, as observed when replaying the saturated signal, is displayed in Fig. 4(b) and the assumed actual magnetization of the tape is obtained from this by graphical integration. (We neglected again such effects as the alteration of field-strength as a consequence of the presence of the high-permeability material of the replay head, the gap and other losses of the replay head, the phase distortion of the replay channel which was not properly phase compensated.)

By this method, in spite of its shortcomings, we obtained a simple tool to tell whether or not the tape is saturated by the bias. In Experiment 2(iii) we replayed the 'optimum' bias through a wide-band amplifier and observed that the signal exhibits the characteristics of a saturated waveform. (The resolution of the iron-powder method is insufficient to observe the double lines.) The experiments, even in this primitive form, provided important backing for the theory. They proved that:

(a) The bias is recorded on the tape and it is justifiable to approximate the waveform of the magnetization to a trapezoid.

(b) the peak value of the 'optimum' bias is near to the upper knee—the saturation point—of the recording medium.

(c) the replay output voltage increases with increasing bias frequency as predicted by the theory.

Conclusion

The theory, which considers the magnetic recording with a.c. bias as a kind of pulse-width modulation process, renders plausible certain features of biased recording for which there had been no satisfactory explanation in the past. Some simple experiments seem to prove the basic assumptions of the theory, and are in agreement with the predictions of the mathematical analysis.

Acknowledgement

The authors are indebted to Prof. D. Gabor, F.R.S., for his helpful criticism of the manuscript.

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Active Low-Pass RC Filters

ASYMPTOTIC CUT-OFF FREQUENCY RESPONSES FROM CASCADED SECTIONS

By D. P. Franklin*

SUMMARY. *By arranging the response of each section of a cascade to provide the contribution of a different pair of conjugate poles in a desired network function, the overall response may be synthesized using simple RC ladders and buffer amplification.*

In a large number of applications involving circuits with high-frequency cut-off, the achievement of large attenuation at finite frequencies is less important than maintaining a desired response within the pass-band. Accordingly, it is sufficient if the response approaches asymptotically

$F(\omega) = A\omega^{-n}$ where A is a constant and n is a positive integer.

Typical of such circuits are constant-*k* filters and transmission lines, *RC* ladders and cascaded broad-band amplifiers.

Such responses can usually be achieved by the combination of inductance and capacitance in resistively-terminated networks, but the results often fall short of the non-dissipative ideal if the cut-off frequencies involved fall below about 100 c/s. The difficulty arises of obtaining a high enough ratio of reactance to resistance in the inductive elements.

Improvement may be obtained by shifting all the poles (and zeros in the general case) in the complex frequency plane by an equal amount parallel to the real axis. A finite insertion loss results, but much lower *Q*s can be tolerated.

To avoid these difficulties, it is common practice to employ networks made up of resistance and capacitance only, losses in the latter usually being negligible, particularly at low frequencies. An extension of this practice is to combine such networks with active elements (linear amplifiers) in feedback circuits and so to achieve results comparable with those obtained at higher frequencies from *LC* combinations[1].

For example, the parallel-T feedback circuit is well known for providing a response equivalent to a simple *LC* band-pass filter[2]. A more generalized discussion has been given, for example, by Kuh and Pederson[3].

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TABLE 1

$[a + bx^2 + cx^4 + dx^6 + ex^8 + fx^{10}]^{1/2}$

where $x = \omega/\omega_0$ for the equal ripple and maximally-flat cases and $x = \omega T$ for maximally-flat group delay.

Order	a	b	c	d	e	f	
1	1	0·01	0	0	0	0	Equal ripple magnitude
2	1·01	−0·04	0·04	0	0	0	
3	1	0·09	−0·24	0·16	0	0	
4	1·01	−0·16	0·8	−1·28	0·64	0	
5	1	0·25	−2	5·6	−6·4	2·56	
1	1	0·01	0	0	0	0	Maximally-flat magnitude
2	1	0	0·01	0	0	0	
3	1	0	0	0·01	0	0	
4	1	0	0	0	0·01	0	
5	1	0	0	0	0	0·01	
1	1	1	0	0	0	0	Maximally-flat group delay
2	1	1/3	1/9	0	0	0	
3	1	1/5	2/75	1/225	0	0	
4	1	1/7	3/245	2/2205	1/11025	0	
5	1	1/9	4/567	1/2835	1/59535	$1/945^2$	

Given the pole locations appropriate to the desired response, a method will be described of setting up a cascade of simple structures to realize that response.

Typical Characteristics

The general expression for the amplitude response with respect to frequency of low-pass networks of the type described is

$$|A(j\omega)| = \sqrt{\frac{1}{1 + a\omega^2 + b\omega^4 + \ldots. + q\omega^{2n-2} + r\omega^{2n}}}$$

where $a, b, c \ldots.$, q, and r are real quantities.

The denominator is, of course, our main interest and for convenience we will define a loss function which is the inverse of the above gain

$$L(\omega) = (1 + a\omega^2 + b\omega^4 + \ldots. + q\omega^{2n-2} + r\omega^{2n})^{\frac{1}{2}}$$

(N.B. For a passive network, $L(\omega) > 0$, and with a low-pass *RC* ladder the coefficients are positive. Any real root for ω^2 must be negative, and complex roots occur in conjugate pairs in the left-hand half of the complex frequency plane.)

Table 1 sets out expressions for $L(\omega)$ for three important classes of characteristic: equal-ripple magnitude (Chebycheff), maximally-flat magnitude (Butterworth)[4] and maximally-flat group delay (Thomson)[5, 6]. For the purpose of illustration, the first two have been so chosen that, when $\omega = \omega_0$, the loss is $1{\cdot}01^{\frac{1}{2}}$ (i.e., 0·043dB) while the third has a general normalizing time constant T. The quantity in the brackets may be considered as a polynomial in $(k\omega)^2$ and factorized, conjugate pairs of complex roots not being separated.

The result of such factorizing is shown in Table 2 for the expression given in Table 1; in each case the frequencies have been normalized to give a coefficient of unity to the term of highest power. It follows that if a cascade of amplifiers, each of which imposes negligible loading on its predecessor, is arranged to provide each of the loss function factors, the desired loss function will apply to the overall cascade.

Suitable Networks

Examining Table 2 we see that there are two forms that the loss function of a given section may take

$$[1 + T^2\omega^2]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

and

$$[1 + KT^2\omega^2 + T^4\omega^4]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The first will be recognized as that of a simple *RC* section (Fig. 1) and has two asymptotic parts—level at unity transmission for low frequencies and rising at 6 dB/octave for high frequencies. In the second, $2 > K > -2$ for, if $K \nless 2$, the expression could be further factorized into two terms of the form of Equ. (1) and if $K \ngtr -2$ real roots would result and the system would be oscillatory.

The main problem, then, is to obtain a circuit configuration corresponding to Equ. (2). At and above

TABLE 2

$[A\{1 + x^2/a^2\}\{1 + b\,x^2/c^2 + d\,x^4/c^4\}\{1 + e\,x^2/f^2 + g\,x^4/f^4\}]^{1/2}$

Where $x = \omega/\omega_0$

Order	A	a	b	c	d	e	f	g	
1	1	10	0	—	0	0	—	0	Equal ripple magnitude
2	1·01	∞	−0·199	2·242	1	0	—	0	
3	1	1·175	−1·351	1·459	1	0	—	0	
4	1·01	∞	−1·741	1·236	1	0·806	0·906	1	
5	1	0·636	−0·589	0·867	1	−1·879	1·143	1	
1	1	10	0	—	0	0	—	0	Maximally-flat magnitude
2	1	∞	0	3·162	1	0	—	0	
3	1	2·154	−1	2·154	1	0	—	0	
4	1	∞	−1·414	1·778	1	1·414	1·778	1	
5	1	1·585	−1·618	1·585	1	0·618	1·585	1	
1	1	10	0	—	0	0	—	0	Maximally-flat group delay
2	1	∞	1	10·05	1	0	—	0	
3	1	10·40	0·0945	11·39	1	0	—	0	
4	1	∞	−0·459	12·81	1	1·671	11·44	1	
5	1	12·19	−0·809	14·24	1	1·149	12·63	1	

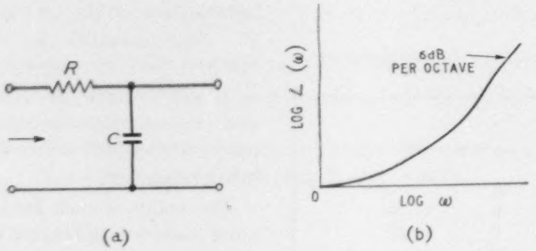

Fig. 1. *A simple RC section (a) and the transmission characteristics (b)*

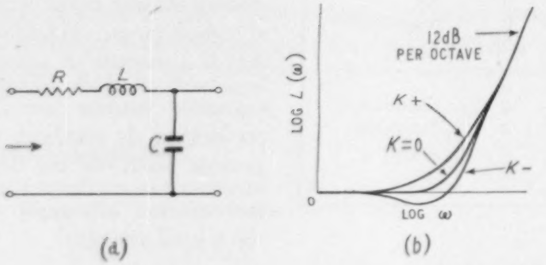

Fig. 2. *The simple RC section with a series peaking inductance added (a) and the characteristics (b)*

medium audio frequencies this can be provided by series 'peaking' inductance as shown in Fig. (2), but at lower frequencies, where inductance is cumbersome or otherwise undesirable, *RC* circuits are preferable.

The two-section *RC* ladder of Fig. (3) gives a loss of the form

$$\left[1+\frac{m^2+n^2+2m+2n+1,}{mn}\cdot T_1{}^2\omega^2+T_1{}^4\omega^4\right]^{\frac{1}{2}}$$

$$\text{where } T_1=\left(\frac{m}{n}\right)^{\frac{1}{2}}RC$$

but the value for *K* exceeds 2 for all positive *m* and *n*. To obtain useful values the use of active circuits involving feedback is indicated.

For the circuit shown in Fig. 4, the ratio of output to input is, in operational form

$$\frac{v_o}{v_i}=\frac{G}{1+G+p\left(\dfrac{m+n+1}{n}\right)T+p^2\dfrac{m}{n}\,T^2}$$

$$\text{where } T=RC$$

$$=\frac{G}{1+G}\;\frac{1}{1+p\,\dfrac{m+n+1}{n(1+G)}\,T+p^2\,\dfrac{m}{n(1+G)}\,T^2}$$

Hence

$$\begin{aligned}L(\omega)&=\frac{1+G}{G}\cdot\left\{1+\left[\frac{(m+n+1)^2}{n^2(1+G)^2}-\frac{2m}{n(1+G)}\right]T^2\omega^2\right.\\&\qquad\left.+\frac{m^2}{n^2(1+G)^2}\,T^4\omega^4\right\}^{\frac{1}{2}}\\&=\frac{1+G}{G}\cdot\left\{1+\left[\frac{(m+n+1)^2}{mn}\,\frac{1}{1+G}-2\right]T_2{}^2\omega^2\right.\\&\qquad\left.+T_2{}^4\omega^4\right\}^{\frac{1}{2}}\quad\cdots\quad\cdots\quad\cdots\quad\cdots\;(3)\end{aligned}$$

where

$$T_2=\left(\frac{m}{n(1+G)}\right)^{\frac{1}{2}}RC$$

By choosing values for *m*, *n* and *G* we can adjust the coefficient *K* of $T_2{}^2\omega^2$ in Equ. (3), and in particular, by making *G* large, we can make the value approach −2 as closely as we wish. However, any change in one of these brings about a corresponding change in T_2. In principle, *m* and *n* could be varied proportionately leaving T_2 constant while modifying *K*, but the trimming of capacitance is most inconvenient.

An alternative circuit is shown in Fig. 5 in which it is assumed that the output impedance of the amplifier is very low[7]. In operational form the output/input ratio is

$$\frac{v_o}{v_i}=\frac{1}{1+p\left(\dfrac{m+n+1}{n}-G\right)T+p^2\dfrac{m}{n}\,T^2}$$

yielding

$$\begin{aligned}L(\omega)&=\left\{1+\left[\frac{(m+n+1-nG)^2}{mn}-2\right]T_1{}^2\,\omega^2\right.\\&\qquad\left.+\,T_1{}^4\,\omega^4\right\}^{\frac{1}{2}}\quad\cdots\quad\cdots\quad\cdots\quad\cdots\;(4)\end{aligned}$$

It will be seen that, once again, by choosing values for *m*, *n* and *G* we can adjust the damping coefficient *K* of $T_1{}^2\omega^2$ but that, in this case, variation of *G* leaves T_1 unaffected. This will, of course, be a much more convenient arrangement to deal with practically.

Optimum Component Values

To achieve a given value for *K*, any set of values of *m*, *n* and *G* appropriate to Equ. (4) could be adopted, but it is expedient to determine the relation between *m* and *n* which requires the minimum value for *G*. Because it is most convenient to set *n* to yield a standard value for *C*/*n* and then to trim *m*, we will consider first the variation of *m*.

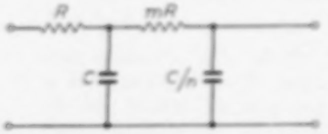

Fig. 3. *A two-section RC ladder*

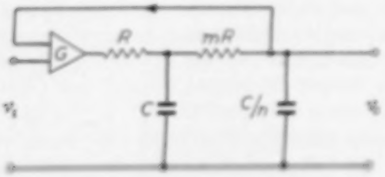

Fig. 4. *The two-section RC ladder with active circuits involving feedback*

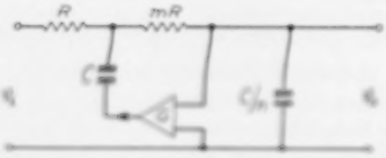

Fig. 5. *An alternative and more convenient arrangement to Fig. 4*

Given the desired K, we have

$$\frac{(m+n+1-nG)^2}{mn} - 2 = K$$

i.e.,

$$nG = m + n + 1 - [mn(K+2)]^{\frac{1}{2}}$$

$$\therefore \quad n\frac{\partial G}{\partial m} = 1 - \tfrac{1}{2}\left[\frac{n}{m}(K+2)\right]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (5)$$

for minimum G

$$\frac{\partial G}{\partial m} = 0$$

$\therefore$ from Equ. (5)

$$m = \frac{K+2}{4}\,n \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

giving

$$T = 2(K+2)^{-\frac{1}{2}}\,T_1 \quad \ldots \quad \ldots \quad (7)$$

and

$$G = \frac{K+2}{4} + 1 + \frac{1}{n} - \frac{K+2}{2} \quad \text{(discarding the negative root)}$$

$$= \frac{2-K}{4} + \frac{1}{n} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

indicating that n (and hence m) should be kept as high as possible.

Suitable Circuit

Let us consider the limiting case of $K = -2$ (just oscillatory); the value of G would then be $1 + (1/n)$. For higher values of K it is, in principle, possible to set a value for n such that $G < 1$.

This immediately suggests a cathode follower or common-collector transistor type of impedance converter for the active element. Furthermore, we will require impedance conversion if we are to connect a number of these circuits in cascade, and we may find it convenient to use the same amplifier stage for both purposes.

Fig. 6 shows such a circuit where the values of K are such that G need be no greater than about 0·75. Transistors VT_1 and VT_2 provide the impedance conversion and VR_1 provides a slight trimming of gain. Capacitor C_3 is made equal to C/n where n has been decided upon using Equ. (8), and R_2 is made such that, in parallel with the effective input resistance of VT_1, it provides a resistance of mR as determined by Equ. (6).

The overall gain at zero frequency is, of course, dependent upon the setting of VR_1 and will normally be as close to unity as possible. When the amplifier G (Fig. 5) has to provide a drop in voltage level as well as impedance this may be obtained by choosing C_1, C_2 and R_1, so that

$$\frac{G(C_1+C_2)}{C_2} = \text{gain at zero frequency.}$$

$$\left.\begin{aligned} C_1 + C_2 &= C \\ R_1 &= R \end{aligned}\right\} \text{as obtained from the basic equations.}$$

In the case where the values of K required involve G having to be uncomfortably close to unity or perhaps somewhat larger, the impedance converter must be altered to allow in-phase gain. Fig. 7 shows such an arrangement in which VT_1 and VT_2 are complimentary types. A current proportional to the voltage at the collector of VT_2 is fed back to the emitter load of VT_1 via preset VR_1, and VT_2 presents a low impedance at its collector. The feedback will also further reduce the emitter impedance of VT_1 and provide a gain close to unity but relatively insensitive to adjustment of VR_1. This point is therefore suitable for providing the output to the next stage.

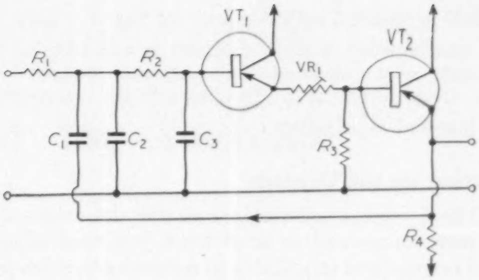

Fig. 6. *Circuit where the values of K are such that G need be no greater than 0·75*

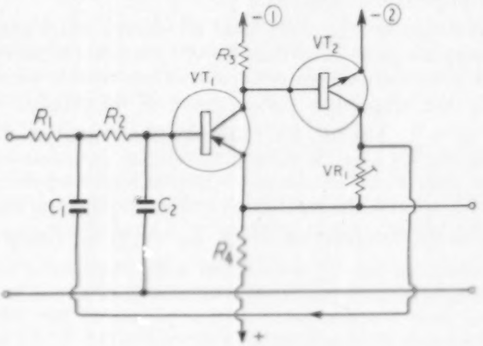

Fig. 7. *Circuit in which the impedance converter can be altered to allow in-phase gain*

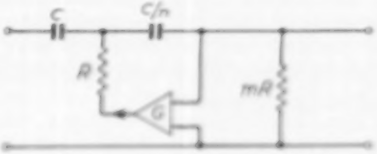

Fig. 8. *The basic circuit of a high-pass section which may be realized in much the same way as for low-pass characteristics*

The output impedance of the impedance converter will normally be finite but small, and therefore departs from the ideal of zero assumed in the derivation of Equ. (4). Generally, it should be small enough for its time-constant with C to be many times smaller than T_1, its effect being to put a 'knee' into the loss function and reduce the slope of the asymptote above this to 6 dB/octave besides slightly modifying the value of T_1 and K from the ideal. The frequency at which the knee occurs is that associated with the above time-constant and should be arranged high enough for there to be adequate loss before the slope falls off.

In cases where the loss function is of odd order in ω^2, we have seen that a pole appears on the negative real frequency axis and gives rise to a factor [Equ. (1)]

which is realized by the circuit of Fig. 1. This simple *RC* combination may be inserted at any position in the cascade which respects the condition of light loading but, if necessary, a further impedance converter may be inserted as a buffer.

Setting up the Cascade

The foregoing calculations enable the nominal value of each component to be decided, but some adjustment will be required in practice to accommodate the production tolerances. Fortunately each of the sections described is isolated from its neighbours by buffer amplifiers and may therefore be adjusted independently to provide the appropriate partial response when connected to a low-impedance signal source.

We saw in Fig. 2(b) that all curves $L(\omega)$ approximately coincide with that for $K = 0$ both for very low and very high values of ω, and it can readily be shown that the departure for a given K is greatest when $T_1\,\omega = 1$. Taking, for example, the circuit of Fig. 6, with R_1, C_1 and C_3 chosen for circuit convenience, we can trim R_2 to obtain the required high-frequency loss with respect to the level for very low frequencies, and this yields the required value for T_1. Next the frequency is set so as to put $T_1\,\omega = 1$ and VR$_1$ trimmed to achieve the appropriate loss, again with respect to the level for very low frequencies. A few points of the response curve may then be plotted in the region $0{\cdot}5 < T_1\,\omega < 2$ to provide checks on experimental error. In particular, when K is negative, the frequency at which the gain is the same as for very low frequencies should correspond to $T_1\,\omega = -K$.

When each partial response has been satisfactorily set up, the sections may be cascaded and the overall response checked. Any shortcomings may usually be eliminated by a final slight trim of the feedback gains of the individual sections.

Conclusion

A method has been given whereby loss functions of the type described may be separated into factors which are then allocated each to a section of a cascade of active *RC* circuits. Transistor circuits have been described but cathode followers may be substituted if desired.

The above process may also be applied to high-pass sections; this is described in the Appendix.

Acknowledgement

The author is indebted to the Directors of E.M.I. Electronics Limited, for permission to publish this article.

APPENDIX

Extension of Argument to High-Pass Response

If in a low-pass loss function $L(\omega)$ we substitute $1/\omega$ for ω, we obtain the loss function for its equivalent high-pass characteristic. This function may be factorized as before and it only remains to find circuit configurations which correspond in their individual losses to those factors.

For the circuits of Fig. 8, following Equ (4),

$$L(\omega) = \left\{1 + \left[\frac{(m + n + 1 - mG)^2}{mn} - 2\right]\frac{1}{T_1{}^2\omega^2} + \frac{1}{T_1{}^4\omega^4}\right\}^{\frac{1}{2}}$$

For a given desired value of K we have as before

$$G = \frac{m + n + 1}{m} - \left[\frac{n}{m}(K + 2)\right]^{\frac{1}{2}}$$

$$\therefore \qquad \frac{\partial G}{\partial m} = -\frac{n + 1}{m^2} + \frac{1}{2}\,\frac{[n\,(K + 2)]^{\frac{1}{2}}}{m^{3/2}}$$

$$\therefore \text{ For minimum } G,\; m^{\frac{1}{2}} = \frac{2(n + 1)}{[n(K + 2)]^{\frac{1}{2}}}$$

$$\text{i.e.,} \qquad m = \frac{4}{K + 2}\,\frac{(n + 1)^2}{n}$$

$$\text{Thus } G_{\min} = 1 + \frac{(n + 1)\,n\,(K + 2)}{4\,(n + 1)^2} - \frac{n\,(K + 2)}{2\,(n + 1)}$$

$$= 1 - \frac{n}{n + 1}\,\frac{(K + 2)}{4}$$

indicating that n (and hence m) should be kept as high as possible.

The basic circuit of Fig. 8 may be realized in much the same way as for low-pass characteristic, the input and output impedances of the impedance converter merging readily with the resistive branches.

If a wide band-pass characteristic is required without frequencies of infinite attenuation in the stop bands, a combination of the high- and low-pass characteristics described may be adopted as required.

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WAYNE KERR, U.K.—GERTSCH, U.S.A. AGREEMENT

A reciprocal sales and manufacturing arrangement has been established between The Wayne Kerr Laboratories Ltd. and Gertsch Products Inc. of Los Angeles, U.S.A.

The agreement provides for the marketing and manufacturing of a wide range of Gertsch electronic instruments in the U.K. by Wayne Kerr and for the sale of Wayne Kerr instruments by Gertsch Products in California, Nevada and Arizona.

Principal Wayne Kerr instruments covered by the agreement include the Transfer Function Computer SA100, transformer ratio-arm bridges, r.f. and v.h.f. bridges, and vibration and distance-measuring equipment.

Some of the Gertsch instruments which will now be marketed by Wayne Kerr include precision ratio transformers, frequency-measuring equipment, synchro and resolver standards and special test instruments.

INTERNATIONAL TELEVISION CONFERENCE

The Electronics and Communications Section of the Institution of Electrical Engineers is organizing an International Television Conference covering all scientific and engineering aspects of television. It is to be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2, from 31st May to 7th June 1962.

CANADIAN AGENCY FOR AVELEY ELECTRIC

Aveley Electric Ltd. have been appointed sole U.K. agents for the Bach-Simpson range of measuring and control instruments. The Bach-Simpson Co. is a completely autonomous Canadian company producing the Simpson Line of instruments entirely from its own resources, but sharing with the Simpson Electric Co. (Chicago, U.S.A.) joint engineering development projects.

The range of instruments which will be available to the British market include multi-range test equipment, panel-mounted control indicators, frequency and running-time meters and oscilloscopes.

MATHEMATICAL TOOLS

By Computer

Newton, Achilles and the Tortoise

In the April 1960 'Tool' we noted that we had the support of no less an authority than Newton in insisting that numbers associated with phenomena occurring in the real world are essentially metrical (associated with a tolerance, because based on measurements which can never be made nor even specified exactly). In this respect, Newton differed radically from his successors.

A correspondent, Mr. H. L. Armstrong, of Queen's University, Kingston, Ontario, has recently brought to our notice a book, entitled *Back to Newton*, by G. de Bothezat, published by Stechert & Co., New York (1936) which is largely in agreement with our ideas that numbers are essentially metrical and, in particular, points out that the famous paradox of Zeno concerning the race between Achilles and the tortoise can best be resolved in terms of metrical numbers.

If we assume that Achilles runs with a uniform speed $10v$ and is initially a distance d behind the tortoise, moving with speed v, then following the race step by step as done by Zeno, the distance of Achilles from his start after the first step, when he is where the tortoise was initially, is d, while the tortoise at the same time is a distance $d/10$ further on; when Achilles has completed this further distance $d/10$, the tortoise is $d/100$ ahead, and so on. The ancient argument was that Achilles therefore never caught up the tortoise. de Bothezat points out that after n steps of this nature, Achilles has gone a distance from his start of

$$D = d + \frac{d}{10} + \frac{d}{10^2} + \ldots + \frac{d}{10^{n-1}}$$
$$= \frac{10d}{9}\left(1 - \frac{1}{10^n}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

while the tortoise is a distance $d/10^n$ ahead. If n is made sufficiently large (de Bothezat uses ∞ as the symbol for this metrical kind of infinity, retaining Ω for 'absolute' infinity, the reciprocal of an 'absolute' zero) both $d/10^n$ and $(10d/9 - D)$ become infinitesimally small quantities (in de Bothezat's language) or what we have called metrical zeros, and therefore Achilles catches the tortoise at a distance $10d/9$ from the start.

As de Bothezat says: "It is curious to think that for more than 2,000 years the human mind was unable to formulate in a fully unambiguous manner the two logical points involved in the Zeno sophism: namely the rigorous conception of 'infinitesimally small' and of 'equality'."

We can sum up the essential ideas of metrical mathematics, discussed fully in earlier 'Tools', as follows. No measurement can be exact (nor indeed could it be specified exactly, since this would require infinite information: specification to an infinite number of significant figures, even if most of these figures were zeros). Eddington pointed out that laws of Nature expressed in the form of 'exact' equality are mere equivalences of our own thinking or even language—alternative names for the same thing. On the other hand, the statement

$$a = b \pm \epsilon$$

where ϵ is small (de Bothezat would call ϵ infinitesimal) has the significant meaning that a and b are interchangeable provided that a tolerance ϵ can be allowed. In classical mathematics, however, a magnificent edifice of exact equivalences is built upon the idea of cardinal numbers. de Bothezat regards this as devoid of logical relation to the world.

Suppose for example that we wish to set up a system of standard resistors adequate for comparison with any resistor likely to be required. We could start with a resistor of convenient magnitude, say of order 1 000 Ω, for which the sole requirement would be stability—insensitiveness to changes of temperature, pressure, location, humidity, etc. The next requirement would be to make a second resistor having as nearly as possible the same resistance as the first, perhaps agreeing to within 1 part in 10^7 for all relevant changes of conditions. Next, a total of 10 such resistors could be made, and the equality of their resistances checked as well as possible. Then we should take it that the 10 resistors in series gave a resistance 10 times that of the original resistor, and that the 10 resistors in parallel gave a resistance one-tenth that of the original resistor, and 10 new resistors each having a resistance equal to that of the first 10 in series, and 10 more each having a resistance equal to that of the first 10 in parallel could then be made, and combined so as to make new standard units having 100 times or 1/100 of the resistance of the original resistor. By the provision of suitable switches we could thus establish a set of 'numbers' for measuring resistances over a range of $10^6 : 1$ but, if we attempted to exceed this range, we should be in danger of error in the last digit of some such 'numbers' since we assumed a stability only to within 1 part in 10^7.

In a digital computer, however, counting can be carried out without error over a much wider range than this; the essential difference here is that a digital computer works by 'quantization'. The pulses which operate such a computer do not have to be identical; the machine will respond identically to pulses of any size within a very broad tolerance.

It is, therefore, our plain duty to make up our mind whether any particular activity, study or endeavour is associated with the metrical world of Newton in which every number is associated with a tolerance because it belongs to a measurement which is neither exact nor capable of exact specification, or with the quantized world of the digital computer, in which the primary entities are the cardinal numbers, which can be manipulated as if exact; the latter world would perhaps regard Newton as a citizen, but as one of miserable unimportance compared with his successors.

In physics and engineering—essentially studies of inanimate nature—there is nearly always direct addition, which is usually linear (as far as we can measure) but does not involve quantization. Thus, if we consider 10^{15} periods of a spectral light component, the total time occupied by these periods will be the sum of the 10^{15} individual periods which are nearly but not exactly equal, and there will be no quantization. On the other hand, in biological studies, or studies of the behaviour of human beings in large numbers, there is a natural quantization. One seed will be considered as equal to another in experiments designed to determine and produce the best grass for a lawn to be made in a particular location: the seeds are not identical, but such differences as there may be are irrelevant.

Limits and Tolerances

The solution of Zeno's paradox lies in the method of limits and tolerances as we have already seen. This method enables the equivalences of classical mathematics to be made useful in our study of the world. Furthermore, classical mathematics without guidance from the inanimate world becomes divided against itself, and fails to give a self-consistent body of relations. Errors of thought (of which, alas, many have bedevilled the history of physics) creep in, from the parallel postulate onwards. de Bothezat points out that there is only one line through a given point which meets a given line at a point infinitely distant (in the absolute sense for which he uses the symbol Ω, not ∞) in both directions, although there are any number of lines through the point which meet the given line at a metrically infinite distance (for which de Bothezat uses the symbol ∞) on one side or the other. It is no vain repetition to say that by such a metrically infinite distance we mean a distance which is sufficiently large—finite, but as large as we please relative to any other distance in which we happen to be interested.

In the light of the above discussion, let us reconsider the process of differentiation, for simplicity in the form of determining a velocity. The standard procedure is to consider a convenient interval, and determine the average velocity during that interval. Then the calculation is repeated for a smaller interval, beginning or ending at the time in which we are interested, or including this time. We cannot reduce the interval to a logical zero, but we often observe that the average velocity approaches more and more closely to a certain value, say V, when the interval gets smaller and smaller. Then we 'close the limit' and think that, just as a cricketer scores four runs for a boundary stroke without actually having to run them, so we have managed to obtain a velocity V purporting to be an 'instantaneous velocity' without having actually committed the heinous crime of division by a logical zero. But this is really confused thinking. The limit should never in fact be closed. The interval can be arbitrarily small—a metrical zero, but not a logical one. As we have seen, we lose nothing by this apparent restriction, because an absolute or logical zero occurs with zero probability, and takes up no logical space.

These results properly belong to the world of inanimate nature, engineering and physics. Curiously enough, they also belong to the quantized world of biology, human affairs (on the large scale) and digital computers. The only difference here is that the quantity by which we may have to divide is necessarily a multiple of a certain minimum unit (an individual person, seed, pulse, etc.). As long as we are dealing with affairs on a sufficiently large scale for the quantization to be applicable, the amount of logical space occupied by the forbidden logical zero only increases from zero to one in the essentially large total number of quanta concerned. As long as we do not attempt to close the limit, rates of change of quantities associated with legitimate quantization are just as valid and useful as those associated with the supposedly 'continuous' phenomena of inanimate nature, which would probably have to be regarded as discrete if we were obliged to think microscopically in terms of distances within the atom.

The gateway to an effective study of the real world is that we can make errors as small as we please, for example in the determination of velocity, if we measure the distance travelled in a sufficiently short time. But although there is such a thing as the average velocity over a small interval, obtained by dividing the distance travelled in the interval by the interval, we really have no right to close the limit and talk about the velocity at a point. The best we can do is to use the Hopkinson-Boltzmann principle of superposition which, in effect, means that for sufficiently small changes of a particular quantity, say time, all associated changes of other quantities of order higher than the first (that is, squares, products, etc., of first-order quantities) are negligible, or that all sufficiently small variations can be regarded as linear. We conclude that as the increments of time get smaller and smaller, the corresponding values of average velocity (or average rate of change) approach a constant, to which our predecessors have found it necessary to give the name of instantaneous velocity. We consider this to be a misleading name, because it suggests that Nature possesses differentiating apparatus, and that it is possible for us actually to make an 'instantaneous' observation instead of merely an observation over a short time-interval.

In the real world (or in the world of legitimate quantization) the above-mentioned "linearity of small things" is universally applicable, and we cannot therefore regard some of the contemporary mathematical processes and functions as relevant.

Our last word is thus, as always: Never forget tolerances and never close limits, and all difficulties of a mathematical nature associated with problems in the real world will be revealed as the artefacts they really are. Mathematics can then become the tool it should be, available to any seeker after truth however limited, instead of the esoteric possession of academic professors —a subject unnecessarily made difficult.

Common-Emitter Amplifier

MEDIUM-FREQUENCY PERFORMANCE

By R. Leek, M.Sc.(Eng.), A.C.G.I., Graduate I.E.E.[*]

SUMMARY. *The conditions are examined under which a simple equivalent circuit for a transistor may be applied with accuracy to an analysis of the common-emitter amplifier. The relationship between the voltage gain of the amplifier and the frequency is then studied theoretically and shown to compare favourably with experimental results. The factors which affect the nature of this relationship are studied and formulae are derived in terms of circuit parameters which indicate the amount of emitter feedback resistance required to provide the maximum bandwidth at any specified zero-frequency voltage gain. This leads to a formula in terms of transistor parameters which enables the best transistor to be selected from a number of possible types when the maximum bandwidth is required at a specified zero-frequency voltage gain.*

It is shown that the theory for the isolated amplifier stage may be applied to amplifiers having cascaded common-emitter stages. The relationship between zero-frequency voltage gain and bandwidth of the cascaded stage is examined theoretically and the results are verified by experiment.

In order to provide satisfactory agreement with practical results obtained over a wide range of frequency with junction transistors, elaborate equivalent circuits for the transistor have been devised[1,2,3,4]. In many applications the transistor is not used in circuits operated at the higher frequencies and simpler equivalent circuits may be employed[5].

The principal objects of this investigation were to examine the limitations of the use of a very simple equivalent circuit for the transistor and, having found the conditions under which this circuit may be applied with accuracy, to establish the principal features of the relationship between voltage gain and bandwidth for the common-emitter amplifier and the factors which control this relationship.

The equivalent circuit chosen to represent the transistor was the simple one shown in Fig. 1. The parameters shown in this circuit may be obtained from the static characteristics for the transistor by considering infinitesimal changes in voltage and currents at the working point. For OC71 transistors the emitter resistance r_e and the base resistance r_b commonly have values of about 20 Ω and 700 Ω, while the collector resistance r_c and capacitance C_c are 500 kΩ and 40 pF, respectively, and the value of the current gain is about 0·98.

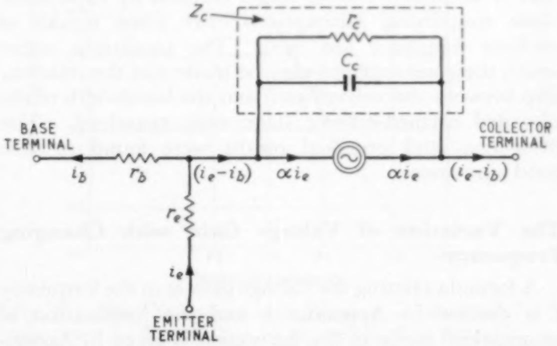

Fig. 1. *Equivalent circuit of transistor*

The only frequency-dependent parameter in the equivalent circuit is taken to be α and it is supposed that the variation of α with changing frequency f is of a form controlled by a single time-constant. If the zero-frequency value of α is α_0 and the frequency at which the magnitude of α falls to 3 dB below its zero-frequency value is f_α, the relationship between α and f is,

$$\alpha = \frac{\alpha_0}{1 + jf/f_\alpha} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

It has been shown[1] that Equ. (1) provides good agreement with practical results up to a frequency of f_α but, for higher frequencies, the magnitude of α is less and the phase angle of α is appreciably more than that prescribed by Equ. (1). For the OC71 transistor the value of f_α is about 1 Mc/s.

The Common-Emitter Amplifier

The basic circuit of the common-emitter amplifier used is shown in Fig. 2. A transistor VT has a collector load R_c, an emitter load comprising R'_e and a resistor R_3 across which was connected a decoupling capacitor C_1. The base can be biased by any convenient method and, in the circuit shown, the resistors R_1 and R_2 were used to establish the necessary base-bias conditions. The input signal was obtained from a signal generator

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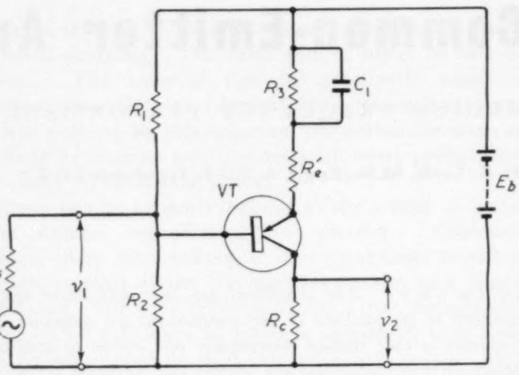


Fig. 2. *Circuit diagram of common-emitter amplifier*

represented by an alternating-voltage source e_s and a source resistance R_s. A direct-voltage supply E_b was adjusted to provide a desired quiescent working point on the transistor characteristic curves. When OC71 transistors were used, the working point was maintained throughout at a point corresponding to a collector current of 1 mA, a voltage on the collector with respect to the emitter of -5 V and a base current of 20 μA.

LIST OF SYMBOLS

$a = \dfrac{R_c}{r_b}$
$a' = \dfrac{R_c}{r_b + R_c'}$
α = current gain factor
α_0 = zero frequency value of α
$b = \dfrac{R_e}{r_b}$
$b' = \dfrac{R_e}{R_b + R_c'}$
$c = \dfrac{r_b}{r_c}$
$c' = \dfrac{r_b + R_c'}{r_c}$
C_1 = decoupling capacitor
C_c = collector capacitance
$d = a + b + ab$
$d' = a' + b' + a'b'$
$= \dfrac{R_b}{r_b}$
E_b = e.m.f. of supply battery
e_s = e.m.f. of alternating source
f_α = bandwidth of α
f = frequency
f_B = bandwidth of isolated stage
f_B' = bandwidth of cascaded stage
f_{B0} = overall bandwidth of multistage amplifier
$f_c = \dfrac{1}{2\pi C_c r_b}$
i_b = instantaneous value of base current
i_e = instantaneous value of emitter current
m = voltage gain of isolated stage
m_0 = zero frequency value of m
m' = voltage gain of cascaded stage
m_0' = zero frequency value of m'
m_T = voltage gain of multistage amplifier
m_{T0} = zero frequency value of m_T
m_{0m}' = maximum value of m'_0
r_b = incremental base resistance
r_c = incremental collector resistance
r_e = incremental emitter resistance
R_1, R_2 = base-bias resistors
R_3 = decoupled-emitter circuit resistance
R_{b1}, R_{b2}.. = base-bias circuit resistance
R_{c1}, R_{c2}.. = collector resistances
$R_c' = \dfrac{R_b R_c}{R_b + R_c}$
R_{c0} = value of R_c which provides m_{0m}'
$R_e = R_e' + r_e$
R_e' = emitter resistance external to transistor
R_{e0} = optimum emitter resistance for maximum bandwidth at specified value of m_0
R_s = resistance of source
v_1, v_2 .. = instantaneous voltages in amplifier circuit
v_{01}, v_{02} .. = instantaneous voltage of Thévenin generator in collector output circuit
$x = \dfrac{f_\alpha}{f_c}$
$y = \dfrac{f}{f_\alpha}$
Z_c = impedance of r_c and C_c in parallel
Z_0 = output impedance of amplifier stage

Scope of this Investigation

A set of measurements was taken of the magnitude of the voltage gain of an earthed-emitter amplifier employing a transistor whose equivalent circuit parameters were known. Some measurements were also made of the phase angle of the voltage gain. The values of collector load resistance R_c, emitter feedback resistance R_e and the frequency were varied over a wide range. The shape of the curves showing the variation of voltage gain with changing frequency was then examined and compared with those expected from a theory based on the simple equivalent circuit for the transistor shown in Fig. 1.

The nature of the relationship between voltage gain and bandwidth was then examined and the principal factors which control the practical limitations of operation were found and verified by the practical results. It was found that an optimum value of emitter feedback resistance is required to provide the maximum voltage gain–bandwidth product at any particular value of voltage gain or bandwidth and formulae for calculating the resistance and the maximum gain–bandwidth product are obtained.

The study was then extended to a consideration of amplifiers comprising cascaded identical common-emitter amplifiers having stages coupled by capacitors. Some simplifying assumptions exist when signals of medium frequency are used. The conditions under which these assumptions may be made and the relationship between the voltage gain and the bandwidth of the cascaded earthed-emitter stage were examined. The theoretical and practical results were found to give good agreement.

The Variation of Voltage Gain with Changing Frequency

A formula relating the voltage gain m to the frequency f is derived in Appendix 1 and the justification of assumptions made in the derivation is given in Appendices 2 and 3. It was found to be convenient to define quantities in terms of parameters shown in Figs. 1 and 2, as follows

$$\left.\begin{array}{l} a = \dfrac{R_c}{r_b},\ b = \dfrac{R_e}{r_b},\ c = \dfrac{r_b}{r_c},\ d = a + b + ab \\ f_c = \dfrac{1}{2\,\pi\,C_c\,r_b},\ x = \dfrac{f_\alpha}{f_c}\ \text{and}\ y = \dfrac{f}{f_\alpha} \end{array}\right\} \quad \ldots \quad (2)$$

Then,

$$m = \frac{-\alpha_0 a}{1 - \alpha_0 + b + cd + jy\,[1 + b + d\,(c + x)]} \qquad (3)$$

Equ. (3) indicates that the modulus and phase angle of m vary with changing frequency in a manner prescribed by a single time-constant. A graph of the modulus of m plotted against frequency should tend to an asymptote of 6 dB per octave at high frequencies and when the value of m is 3 dB below its zero frequency value, the phase angle of m should be 135°. A comparison of the values for the zero frequency voltage gain obtained from Equ. (3) and by experiment follows in a later section (The Relationship between Zero Frequency Voltage Gain and Bandwidth). Here, the shapes of the curves of modulus and phase angle of m plotted against frequency obtained from Equ. (3) are compared with those found by experiment. It is convenient to normalize the values of m with respect to their zero-frequency values and the values of frequency with respect to the bandwidth f_B. The results are drawn together as shown in Fig. 3. The four curves are drawn showing the results obtained with a small and a large value of R_e for both a low and a high value of zero-frequency gain. A transistor having the parameters indicated in Table B was used, the value of f_α being 650 kc/s. All four curves follow the shape of the line 5 calculated from Equ. (3). The results indicate that, as far as the modulus of the gain is concerned, Equ. (3) provides agreement within experimental error. This agreement holds up to a frequency of at least 800 kc/s.

Some measurements were made of the phase angle of m and these are shown plotted against frequency as curves A and B in Fig. 4. The error in measurement of the phase angle was within ± 10%. The theoretical values of phase angle, obtained from Equ. (3), are also plotted in Fig. 4 and give the curves X and Y. Although the accuracy of measurement of the phase angle was not high, it is clear that the simple relationship provided by Equ. (3) gives good agreement up to a frequency of 200 kc/s but, at higher frequencies, the phase angle becomes progressively less than that specified by Equ. (3). This situation was envisaged in Appendix 3 where it was shown that the omission of certain frequency-dependent terms would have a negligible effect on the modulus of the voltage gain but that the phase angle would be in error, particularly at the higher frequencies. The phase angle was therefore calculated using the more accurate Equ. (24) and ignoring only the second term in the numerator ; this gave curves X′ and Y′. The extra phase shift introduced by including the second term in the numerator at a frequency of 650 kc/s gave point P. Finally, a more accurate formulae than Equ. (1) relating the current gain α to the frequency f was used[1],

$$\alpha = \frac{\alpha_0}{\cosh\ \sqrt{j2{\cdot}43y}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

When the frequency was 650 kc/s, and the second term in the numerator was omitted, Equs (24) and (4) gave point Q in Fig. 4.

It is concluded that the simple equivalent circuit of

TABLES A and B

TRANSISTOR PARAMETER	TABLE A	TABLE B
r_e (Ω)	24 ± 10 %	20 ± 10%
r_b (Ω)	770 ± 5 %	1,240 ± 5%
r_c (MΩ)	0·500 ± 10 %	2·70 ± 10%
α_0	0·980 ± ½ %	0·986 ± ½%
f_α (kc/s)	980 ± 5 %	650 ± 5%
C_c (pF)	40 ± 10 %	33 ± 10%

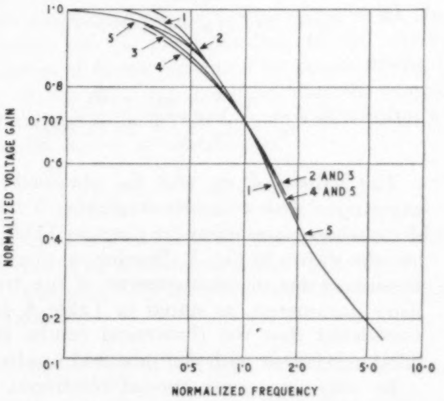


Fig. 3. *Voltage gain plotted against frequency on normalized scales; curve 1, $R_e = 20\ \Omega$, $m_0 = 29$, curve 2, $R_e = 820\ \Omega$, $m_0 = 31$; curve 3, $R_e = 20\ \Omega$, $m_0 = 3{\cdot}5$, curve 4, $R_e = 820\ \Omega$, $m_0 = 3{\cdot}5$; curve 5, follows law of single time-constant*

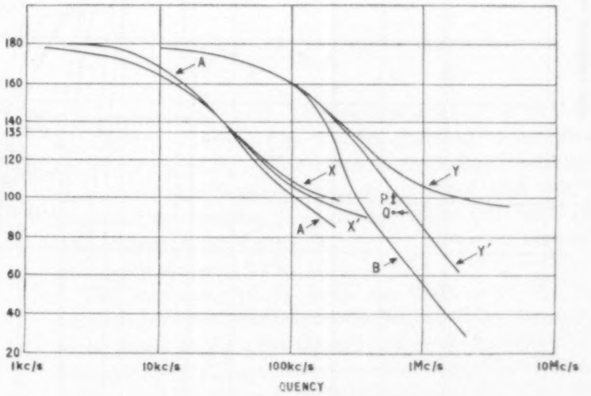


Fig. 4. *Phase angle of voltage gain plotted against frequency; curve A, experimental curve = 67 Ω, m_0 = 15·5; X and X′, theoretical curves = 67 Ω, m_0 = 15·5; B, experimental curve = 700 Ω, m_0 = 1·2; Y and Y′, theoretical curves = 700 Ω, m_0 = 1·2*

Fig. 1 and the Equ. (3) are adequate to provide good agreement between theoretical and practical values of the phase angle of voltage gain up to a frequency of 200 kc/s. Between frequencies of 200 kc/s and 2 Mc/s, the exact equations provide values about 20° too large and the use of a more complex equivalent circuit for the transistor than that shown in Fig. 1 is indicated.

The Relationship Between Zero-Frequency Voltage Gain and Bandwidth

Having established that the modulus of the voltage gain at medium frequencies may be accurately found from Equ. (4), a study was made of the relationship between the modulus of the zero-frequency voltage gain m_0 and the frequency for which the gain fell 3 dB below its zero-frequency value. This latter frequency was called the bandwidth f_B.

It follows by inspection from Equ. (3) that the zero-frequency voltage gain is negative which accounts for the phase reversal of a signal on passing through the amplifier. However, the modulus is positive.

$$m_0 = \frac{\alpha_0 a}{1 - \{\alpha_0 + b\} + cd} \quad \cdots \quad (5)$$

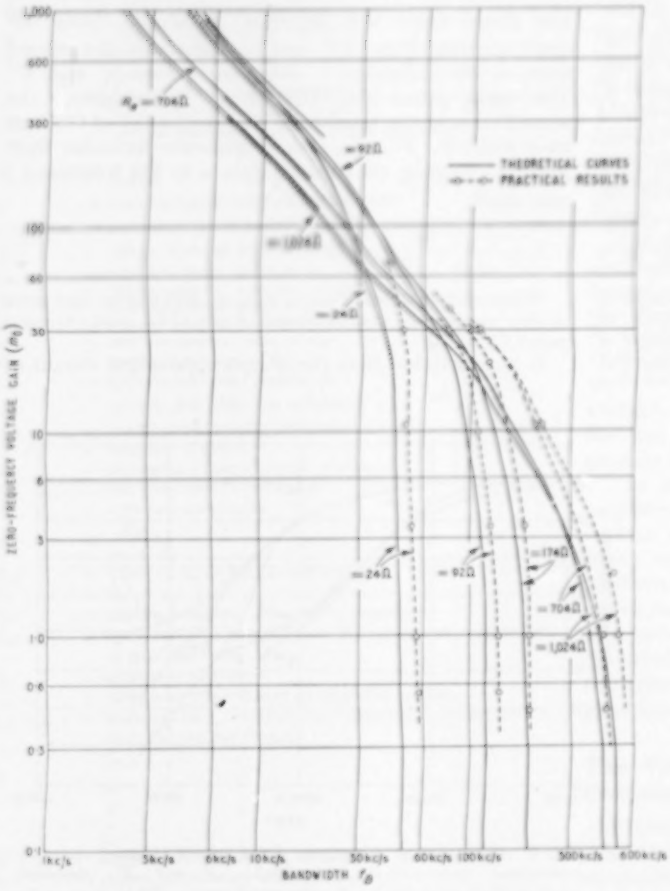


Fig. 5. *Zero-frequency voltage gain plotted against bandwidth*

The equation for f_B is also obtained by inspection from Equ. (3),

$$f_B = f_\alpha \cdot \frac{1 - \alpha_0 + b + cd}{1 + b + d\,(c + x)} \quad \cdots \quad (6)$$

When a particular transistor is used in the amplifier circuit, the only variables present are the collector load resistance R_c and the emitter feedback resistance R'_e. The relationship between m_0 and f_B might be studied by drawing a graph of m_0 against f_B for each of a range of values of R_e or of R_c. The range of values of R_e would give one set of graphs while the range of values of R_c would give another set of graphs. If the two sets of graphs were plotted on the same sheet, a grid would appear and the values of R_e and R_c at any point in the (m_0, f_B) plane covered by the grid could be read off. As an illustration of the procedure, a set of graphs will be obtained for a range of values of R_e. The equation for each graph in this set involves m_0, f_B, R_e and transistor parameters and is found from Equs (5) and (6) by eliminating R_c as indicated in Appendix 4, Equs (30) and (31). These equations were used to draw a set of curves for a range of values of R_e as shown in Fig. 5 where logarithmic scales are used for both axes and values for transistor parameters are taken from Table A.

In evaluating Equs (30) and (31) it is observed that owing to the relatively large value of the resistance r_c, the terms in c do not greatly affect the results. Putting c equal to zero in Equs (30) and (31) produces the dotted lines in Fig. 5 for R_e equal to 24 Ω and 1,024 Ω. It is seen that the curve for the smaller value of R_e is in error by a greater amount, particularly when the value of m_0 is large, and f_B is about 10% too small when m_0 is 200. The theory given in the Appendices is applicable with accuracy for all values of m_0 but in the main text the simplified results will be given, putting $c = 0$. These simplified results may be accurately applied under all normal working conditions of the amplifier. From Equs (31) and (30), putting $c = 0$, and using Equ. (2),

$$f_B = \frac{\alpha_0 f_m}{\alpha_0 + 2\pi\, C_c f_m\, m_0\, (r_b + R_e)} \quad (7)$$

where

$$f_m = f_\alpha \cdot \frac{r_b\,(1 - \alpha_0) + R_e}{r_b + R_e\,(1 + 2\pi\, C_c\, f_\alpha r_b)} \quad (8)$$

The values of m_0 and f_B, obtained by experiment with a circuit employing a transistor whose parameters are given in Table A, are also shown in Fig. 5. Bearing in mind the possible errors in measurement of the transistor parameters, as stated in Table A, it is considered that the theoretical results give good agreement with the practical results.

By imposing some special conditions on Equs (7) and (8), the properties of these curves may be revealed as follows with reference to Fig. 6 where m_0 is again plotted against f_B on logarithmic scales.

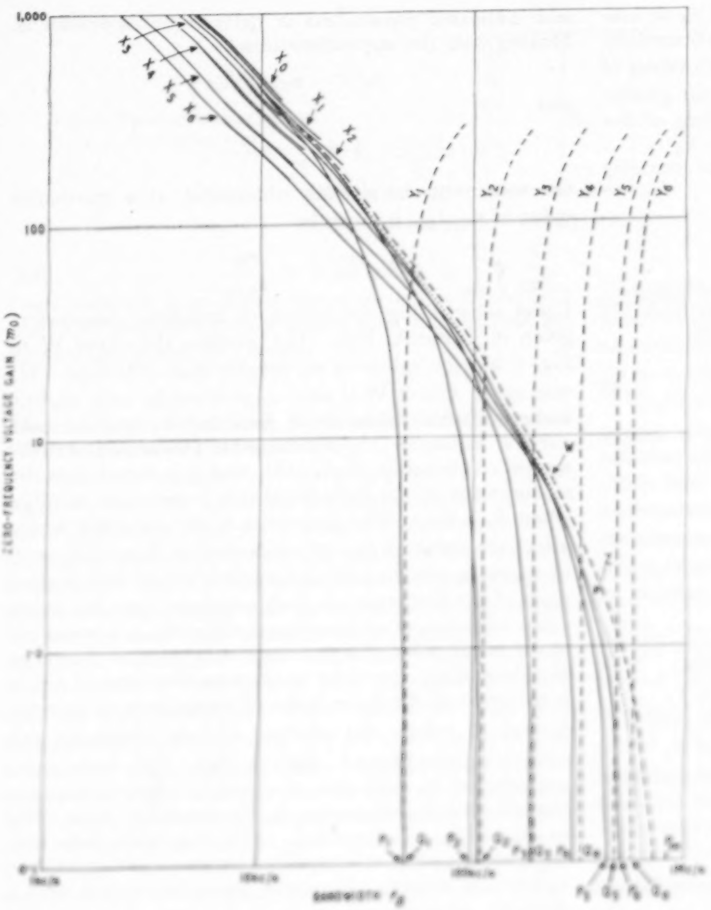


Fig. 6. *Theoretical curves*

(i) m_0 *Tends to Zero*

When m_0 is very small, the denominator of Equ. (7) falls towards unity and the bandwidth approaches the value f_m. Each value of R_e will provide its own value of f_m which is therefore the maximum bandwidth obtainable for the particular value of R_e. In Fig. 6, the values of f_m corresponding to the chosen range of values of R_e are indicated by points P_1, P_2, P_3, etc.

From Equ. (8) it follows that the magnitude of f_m increases as R_e is raised up to a maximum which occurs when $R_e \rightarrow \infty$ whereupon

$$f_m = \frac{f_\alpha}{1 + 2\pi C_c f_\alpha r_b} = \frac{f_\alpha}{p}, \text{ say,} \quad \ldots \quad \ldots \quad (9)$$

where p is a constant for the transistor. This maximum value of f_m is shown as point P_m in Fig. 6.

For an OC71 transistor with parameters as shown in Table A,

$$f_m = \frac{f_\alpha}{1{\cdot}19} = 823 \text{ kc/s.}$$

The maximum bandwidth obtainable at very low gains therefore occurs with large emitter feedback resistance and its value is about 85% that of the current gain bandwidth α.

(ii) m_0 *Tends to Infinity*

It was shown above that Equs (7) and (8) may be used with accuracy instead of Equs (30) and (31) except at high values of m_0. In order to study the effect of high values of m_0 on the shape of the curves of Fig. 6, it is necessary to put $m_0 \rightarrow \infty$ in Equs (30) and (31). It is shown in Appendix 4 that

$$m_0 f_B \rightarrow \frac{\alpha_0}{2\pi C_c (r_b + R_e)} = m_0 f_{BC}, \text{ say,} \qquad (10)$$

f_{BC} is the value of f_B at high values of m_0 when the reactance of the collector capacitance is the only frequency-dependent parameter in the circuit; i.e., $f_\alpha \rightarrow \infty$. Equ. (10) indicates that the voltage gain-bandwidth product tends to a constant value. The product depends on the value of R_e and on substituting the values of parameters from Table A, Equ. (10) gives the straight-line asymptotes X_1, X_2, X_3 . . . shown in Fig. 6.

When the value of R_e is small compared with r_b Equ. (10) approximates to

$$m_0 f_{BC} = \frac{\alpha_0}{2\pi C_c r_b}$$

This equation provides the line X_0 in Fig. 6 and it is observed that this line is displaced only slightly from the lines X_1, X_2, X_3 . . . . . . corresponding to values of R_e up to about 200 Ω. It may therefore be said that, at high values of m_0, the gain–bandwidth product is improved by reducing the product of the collector capacitance and base resistance.

(iii) f_c *Tends to Infinity*

If the only frequency-dependent parameter in the amplifier is the current gain α, then the product of the collector capacitance and the base resistance must be negligible and the value of f_c tends to infinity. Equs (7) and (8) then become

$$f_B \rightarrow f_m' = f_{B_\alpha}, \text{ say,} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where

$$f_m' = f_\alpha \frac{1 - \alpha_0 + b}{1 + b} = f_\alpha \frac{r_b(1 - \alpha_0) + R_e}{r_b + R_e} \quad \ldots \quad (12)$$

Equations (11) and (12) provide a set of vertical lines Y_1, Y_2, Y_3 . . . . . on Fig. 6 corresponding to the chosen set of values of R_e. The effect of including c will again only have significance when m_0 is large and may be observed by putting $f_c \rightarrow \infty$ in Equs (30 and (31), whereupon lines Y_1, Y_2, Y_3 . . . . . . become curved at the top as shown in Fig. (6).

The variation of f_B with m_0 when α is the only frequency-dependent parameter is called f_{B_α} and is seen to be equal to f_m' over the normal working range of m_0. The values of f_m' are shown as points Q_1, Q_2, Q_3 . . . . . in Fig. 6 which are only slightly displaced from the points for f_m, P_1, P_2, P_3 . . . . . Equ. (12) is somewhat less complicated than Equ. (8) and only a small error is introduced in calculating f_m from Equ. (8). This also shows how the relationship between m_0 and f_B at low values of m_0 is controlled by f_α and is little affected by the presence of the product $C_c r_b$, while at high values of m_0 it was shown, in (ii) above, that $C_c r_b$ had far greater significance than f_α. In fact, the contributions of the lines $X_1, X_2, X_3 \ldots\ldots$ and the lines $Y_1, Y_2, Y_3 \ldots\ldots$ to the complete variation of m_0 with f_B can be concisely stated as shown in the next section.

(iv) The Relationship between f_B, $f_{B\alpha}$ and f_{BC}

It was shown above that f_m may be replaced by f_m'. When f_m' is used instead of f_m in Equs (7), (10) and (11) it is found that the following relationship holds,

$$\frac{1}{f_B} = \frac{1}{f_{B\alpha}} + \frac{1}{f_{BC}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Hence, f_B is related to $f_{B\alpha}$ and f_{BC} by a simple reciprocal law. An examination shows that the law also holds good for the precise Equs (31), (10) and (33). The contributions of the two frequency-dependent parameters, the collector reactance and the current gain to the bandwidth of the amplifier may be likened to the contribution to the total resistance of two resistors in parallel.

A constructional method of quickly obtaining the set of curves relating m_0 and f_B is now apparent. The straight lines $X_1, X_2, X_3 \ldots\ldots$ and $Y_1, Y_2, Y_3 \ldots\ldots$ are drawn as shown in Fig. 6 using Equs (10) and (12) respectively with a suitable range of values of R_e. The values of $f_{B\alpha}$ and f_{BC} taken from corresponding pairs of lines X_r and Y_r are then combined using Equ. (13) and the curves relating m_0 and f_B may be sketched in.

Formula for the Maximum Voltage Gain-Bandwidth Product

It is clear from the curves of voltage gain m_0 plotted against bandwidth f_B in Fig. 6 that, at each value of m_0, the maximum value for the bandwidth is obtained for a particular value of R_e. The optimum value of R_e required for a given value of m_0, R_{e0} say, is found from Equ. (35) in Appendix 5. It is shown that, if m_0 is less than 50, an error of less than 14% is introduced in finding the value of R_{e0} from the simplified formula,

$$R_{e0} = \sqrt{\frac{r_b}{2\pi\, C_c f_\alpha\, m_0}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Using Equ. (14), the optimum value of R_e required to provide the maximum bandwidth, f_{B0} say, for each of a range of values of m_0 has been calculated and the bandwidth corresponding to each of these values of R_{e0} found from Equs (5) and (6). Corresponding pairs of values of m_0 and f_{B0} are plotted in Fig. 6 and the curve Z obtained. The whole family of curves relating m_0 and f_B, obtained by unlimited variation of R_e and R_c, fills up a zone of the (m_0, f_B) plane bounded by a limit locus, line Z. Every curve relating m_0 and f_B for a particular value of R_e touches line Z at some value of m_0 and the value of f_B at the point of contact is the maximum bandwidth obtainable for the value of m_0. Since no restriction is placed on the values used for R_e and R_c, it is logical that the limit-locus equation should contain only transistor parameters, m_0 and f_B.

An expression for the limit locus in terms of m_0, f_B and transistor parameters is derived in Appendix 6. Making only the approximations

$$r_b(1 - \alpha_0) \ll R_{e0}$$

and

$$1 + \frac{1}{\alpha_0} = 2,$$

the maximum bandwidth obtainable at a particular value of m_0, f_{B0}, is given by

$$\frac{1}{f_{B0}} = \frac{1}{f_\alpha}\left(1 + 2\sqrt{m_0\, x}\right) + \frac{m_0}{\alpha_0 f_c} \quad \ldots \quad \ldots \quad (15)$$

Upon substituting the values of transistor parameters given in Table A, Equ. (15) provides the curve W in Fig. 6 but for values of m_0 greater than 100 Equ. (37) was used. Curve W is seen to give results only slightly more optimistic than those provided by the accurate curve Z. Equation (15) is arranged in a form similar to the special relationship, Equ. (13), and it is noted that the second term on the right-hand side is the value of $1/f_{BC}$ when R_e is zero. The properties of the transistor which affect the value of f_{B0} are embodied in Equ. (15) and, on carrying out the calculations, it is found that the two parts of the first term are both necessary over the whole range of values of m_0 considered while the two terms are of the same order of importance over most of the range of values of m_0. In order to increase the value of f_{B0}, it is therefore of the same order of importance to increase f_α and to reduce the product of base resistance and collector capacitance. Also, in Equ. (37) both terms are affected by the value of r_b and in order to examine the effect on f_{B0} of varying r_b, the condition [Equ. (28)] in Appendix 1 is recalled. The first term falls with increasing r_b when r_b is small but increases with increasing r_b when r_b is large. However, since the second term is directly proportional to r_b, it is clear that the best improvement in the value of f_{B0} would result from making r_b as small as possible.

When a common-emitter amplifier is required with a maximum voltage gain–bandwidth product it may be necessary to choose a transistor from several types and there is no one figure of merit which may immediately determine the choice. If the parameters of the transistors are known, Equ. (15) may be used to show the variation of the zero-frequency voltage gain with the bandwidth for each of the transistor types under conditions where the voltage gain–bandwidth product is a maximum.

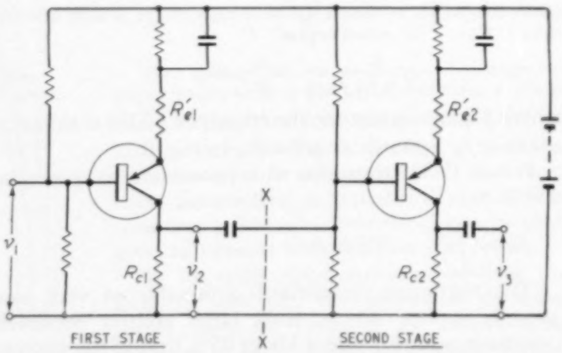
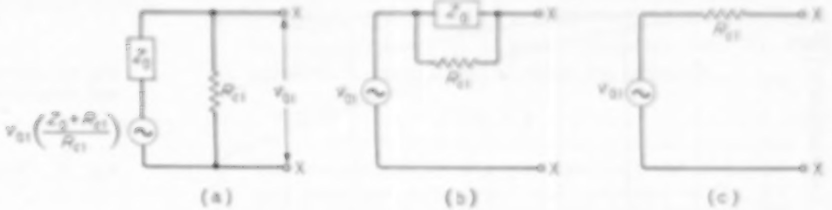


Fig. 7. *Circuit diagram of two coupled common-emitter amplifiers*

Fig. 8. *Thévenin's theorem applied to the collector circuit*



Each transistor type gives a curve like curve W in Fig. 6 and, after comparing these curves with one another, the best transistor may be chosen.

The Voltage Gain and Bandwidth of Cascaded Common-Emitter Amplifiers

When two common-emitter amplifiers are coupled together, the collector of the first stage may be directly connected to the base of the second if correct provision for biasing is made or a coupling capacitor may be used as shown in Fig. 7. The capacitance of this capacitor may be made so large that its reactance at the lowest frequencies used may be taken to be negligible. The amplifiers considered first comprise stages having similar values of emitter and base bias resistance and collector load resistance. Each individual stage will therefore have the same zero-frequency voltage gain and bandwidth when isolated from neighbouring stages. Referring to Fig. 7 it is clear that the ratio v_3/v_2 represents the voltage gain of the isolated stage, while the ratio v_2/v_1 represents the voltage gain of the cascaded stage. The voltage gain and bandwidth of the second stage may be found directly from Equs (5) and (6) but, as far as the first stage is concerned, it is necessary to consider the effect of connecting the second stage. If the circuit were broken at XX, the voltage across XX would rise from v_2 to a value v_{01}, say. If the output impedance of the first stage is Z_0, applying Thévenin's theorem, the circuit across XX may be represented by the diagram Fig. 8 (b). An expression for the output impedance Z_0 is derived in Appendix 7. It was found by experiment that the bandwidth of cascaded stages providing a zero-frequency voltage gain of more than unity did not exceed 120 kc/s so, in this study, the effect of Z_0 on the theory need not be considered above 120 kc/s. Some values of Z_0 have been calculated, using Equ. (37) and the transistor parameters of Table B, and are given in Table C. Although the values of Z_0 are relatively small when the emitter resistance R_e is small, the values of collector load resistance R_c required to provide a given voltage gain are also relatively small.

When the frequency is fixed, the condition that the output impedance Z_0 should be much larger than the value of collector resistance R_c is the same as the condition that the ratio of the voltage gain to the collector load resistance, m/R_c, should be constant for all values of collector load resistance. It is noted that the ratio m/R_c is the value of the current flowing in the collector load resistance R_c when the input voltage is unity. This latter condition was investigated experimentally and graphs showing the ratio m/R_c plotted against R_c for several frequencies and two extreme values of emitter resistance are given in Figs. 9 and 10. It is observed that except when high values of emitter resistance and frequency are used, the values of the ratios m/R_c corresponding to each fixed frequency do not vary greatly. This result would be expected from Equ. (3) from which it is clear that until the values of R_e and R_c are sufficiently large, to make the term in d in the denominator relatively large, the voltage gain is directly proportional to the value of R_c.

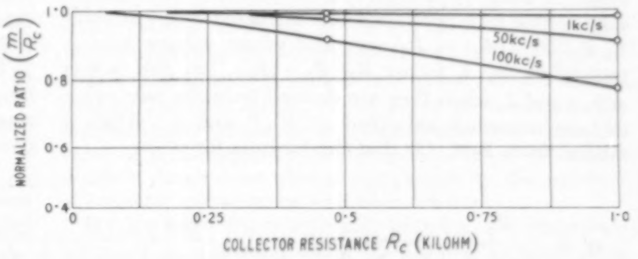


Fig. 9. *Variation of normalized ratio (m/R_c) with changing collector resistance (R_e = 20 Ω)*

TABLE C

FREQUENCY (kc/s)	α	Z_c (kΩ)	R_e = 20 ohms		R_e = 1,020 ohms	
			Z_0 (kΩ)	$\lvert Z_0\rvert$ (kΩ)	Z_0 (kΩ)	$\lvert Z_0\rvert$ (kΩ)
1	$0{\cdot}986 - j0{\cdot}001_5$	$2{,}052 - j1{,}150$	$62{\cdot}6 - j31{\cdot}0$	69·8	$925 - j527$	1,080
50	$0{\cdot}980 - j0{\cdot}075_5$	$3{\cdot}44 - j\ 96{\cdot}4$	$7{\cdot}29 - j3{\cdot}17$	7·95	$6{\cdot}20 - j\ 44{\cdot}5$	44·9
100	$0{\cdot}963 - j0{\cdot}148$	$0{\cdot}86 - j\ 48{\cdot}2$	$7{\cdot}10 - j2{\cdot}38$	7·48	$4{\cdot}94 - j\ 22{\cdot}7$	23·3
200	$0{\cdot}901 - j0{\cdot}277$	$0{\cdot}22 - j\ 24{\cdot}1_5$	$6{\cdot}66 - j2{\cdot}68$	7·18	$4{\cdot}39 - j\ 12{\cdot}2$	13·0

F

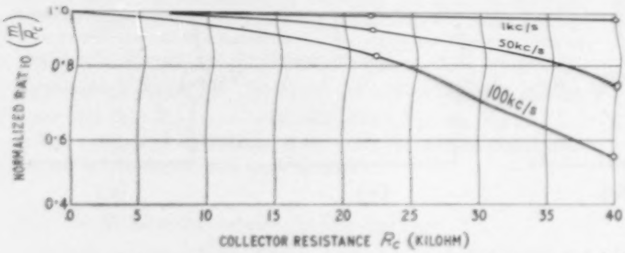
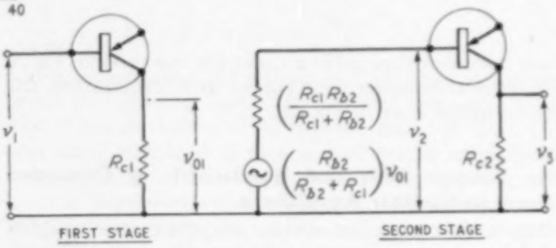


Fig. 10. (*Left*) *Variation of normalized ratio* (m/R_c) *with changing collector resistance* $(R_e = 1{,}020\ \Omega)$

Fig. 11. (*Below*) *Equivalent circuit for coupled stages*


v_1
R_{c1}
v_{01}
$\left(\frac{R_{c1}R_{b2}}{R_{c1}+R_{b2}}\right)$
$\left(\frac{R_{b2}}{R_{b2}+R_{c1}}\right)v_{01}$
v_2
R_{c2}
v_3
FIRST STAGE
SECOND STAGE

It is shown below that a theory based on the assumption that the output impedance is much greater than the collector load resistance is adequate to provide good agreement with experimental results in the whole frequency range considered. The circuit of Fig. 8 (b) then reduces to that of Fig. 8 (c). There are base-biasing resistors in the coupling circuit and, if they are represented by an equivalent resistance R_b between base and earth, again applying Thévenin's Theorem, the circuit of Fig. 7 may be drawn as shown in Fig. 11. Sometimes the value of R_b used in amplifiers is much larger than the value of R_c and the presence of R_b may be neglected, but in general R_b must not be so large that varying leakage currents cause appreciable shift of the working point.

Having arranged the two-stage amplifier in the form shown in Fig. 11, the overall voltage gain of the amplifier, m_r say, may be found using formulae derived for the single stage. It is observed that

$$m_r = \frac{v_2}{v_1} \times \frac{v_3}{v_2} = \frac{v_{01}}{v_1} \times \frac{v_3}{v_{01}} \quad \cdots \quad (16)$$

v_{01}/v_1 is the voltage gain m of a single isolated stage as given by Equ. (3). v_3/v_{01} is the voltage gain of a cascaded stage, m' say, and this may also be found from Equ. (3) since the second stage is effectively an isolated stage whose base resistance r_b is increased by an amount $R_b R_c/(R_b + R_c) = R_c'$ say, and whose voltage gain is multiplied by a factor $R_b/(R_b + R_c)$. Let the factors a, b, c and d, when they are derived from the new value of base resistance be called a', b', c' and d'. Then it follows from Equ. (3) that the formula for m' is

$$m' = \frac{R_b}{R_b + R_c}\left[\frac{-\alpha_0 a'}{1 - \alpha_0 + b' + c'd' + jy\,[1 + b' + d'\,(c + x)]}\right] \quad \cdot\cdot \quad (17)$$

The expression for the overall voltage gain of the two stages becomes

$$m_r = m\,m' \quad \cdots \quad (18)$$

By direct analogy with Equs. (5) and (6), the zero frequency value of m', m_0' say, and the frequency at which m' falls to 3 dB below its value m_0', f_B' say, are obtained by inspection from Equ. (17). The values of transistor parameters given in Table A were used to provide theoretical curves of zero voltage gain m_0' plotted against bandwidth f_B' for the cascaded stage, each curve corresponding to a particular value of R_e as shown in Fig. 12.

An experimental investigation of the voltage gain and bandwidth of two-stage cascaded amplifiers was carried out and the measurements were made using a circuit identical with that of Fig. 7. It will be appreciated that, although the use of the ratio v_3/v_{01} proves convenient in obtaining Equ. (17) for the voltage gain of a cascaded stage from the foregoing theory on isolated stages, this ratio cannot be measured experimentally on the second stage. However, the following argument shows that m', defined above as v_3/v_{01}, can be measured on the first stage. From Equ. (16), v_3/v_2 and v_{01}/v_1 both represent the gain of an isolated stage and the ratio v_2/v_1 is identical with the ratio $v_3/v_{01} = m'$. Values of m' may therefore be obtained experimentally by measuring the ratio v_2/v_1 on the first stage. The practical results for v_2/v_1 are plotted against the measured values of bandwidth of the cascaded stage in Fig. 12 and good agreement with the theoretical is observed. An indication of the values of collector load resistance R_c required is given in Fig. 12 by plotting lines corresponding to constant values of the ratio R_c/R_e or P, say, as shown.

The factors affecting the relationship between zero-frequency voltage gain and bandwidth of the cascaded stage may be examined by imposing some particular conditions on Equ. (17) as follows.

(i) m_0' *Tends to Zero*

The value of m_0' tends to zero when the collector load resistance R_c is very small or very large. When R_c is very small, the equation for f_B' reduces to that for f_B, Equ. (6), and the upper limit of bandwidth f_B' is the same as that for the isolated stage, f_m. The curves of Fig. 12 therefore tend to upper limits of bandwidth similar to those shown for the curves of Fig. 6.

When the value of R_c is very large, since $c'd'$ contains a term in R_c^2, the value of m_0' again falls towards zero.

The significance of the value of R_c may also be appreciated from the fact, shown above, that a cascaded stage acts like an isolated stage with its base resistance increased by an amount R_c'. It is clear that, when the

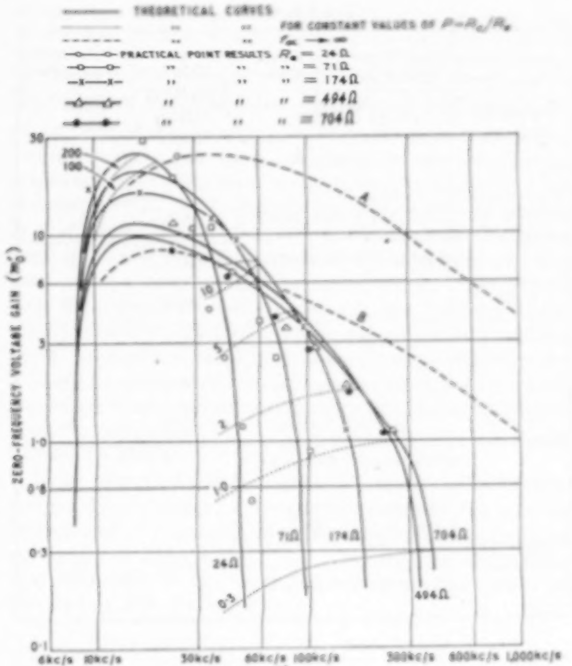


Fig. 12. *Zero-frequency voltage gain plotted against bandwidth of cascaded stage*

value of R_c is small, R_c' is small, and the behaviour of the isolated and cascaded stages will be similar. When the value of R_e is larger, the cascaded stage acts like an isolated stage having large base resistance and consequently low voltage gain.

Regarding the effect of large values of R_e on the value of f_B', attention is drawn to the penultimate sentence of Appendix 1. The lower limit of f_B' to which all the theoretical curves of Fig. 12 converge is not of practical interest.

(ii) m_0' *a Maximum*

For every particular value of emitter resistance R_e there is an optimum value of collector load resistance R_c which provides a maximum value of the cascaded voltage gain m_0'. The optimum value of R_c, R_{c0} say, is found from Equ. (17) as indicated in Appendix 8, although the rather lengthy algebraic manipulation is not given, and the result is Equ. (39). When the base-biasing resistance R_b is large compared with R_c, the value of R_{c0} may be found from the following

$$R_{c0} = \frac{R_e}{b\sqrt{c}}\sqrt{1 - \alpha_0 + b} \qquad \ldots \quad \ldots \quad \ldots (19)$$

This expression for R_{c0} contains only the value of R_e and transistor parameters and, referring to Fig. 12, it enables the value of R_c to be found if the amplifier is required to operate at a point on the peak of a curve of m_0' against f_B'. The value of m_0' corresponding to the peak is the maximum obtainable for the cascaded voltage gain when the particular value of R_e is used. It is now of interest to obtain an expression for the maximum value of m_0,' m_{0m}' say, in terms of R_e and transistor parameters only. This may be done by substituting the expression for R_{c0} obtained above into the equation for m_0' obtained from Equ. (17) with $y = 0$. This substitution gives a cumbersome result when Equ. (38) has to be used and is best calculated step by step. In practice, however, Equ. (19) may more often be used and the substitution leads to the following comparatively simple formula, neglecting only an additional relatively small term $(2\,bc + c)$ which occurs in the denominator.

$$m_{0m}' = \frac{\alpha_0}{1 - \alpha_0 + 2\sqrt{c}\,(1 - \alpha_0 + b)}$$

This equation for m_{0m}', which contains only R_e and transistor parameters, was applied with values taken from Table A and found to give good agreement with the maximum values of m_{0m}' obtained as indicated by the peaks of the curves in Fig. 12.

(iii) f_α *Tends to Infinity*

When the collector reactance is the only frequency-dependent parameter (i.e., $f_\alpha \to \infty$) the relationship between the zero-frequency voltage gain and the bandwidth becomes as shown by the curves A and B in Fig. 12 corresponding to values of 24 Ω and 1,024 Ω respectively. The curves A and B are not so simple as the straight lines X_1, X_2, X_3 . . . of Fig. 6 pertaining to the isolated stage. However, it is again clear that when the value of zero-frequency voltage gain is large, the bandwidth is determined almost completely by the product of collector capacitance and base resistance.

Having found the relationship between zero-frequency voltage gain and bandwidth for isolated and cascaded stages it is shown below that an estimate of the overall

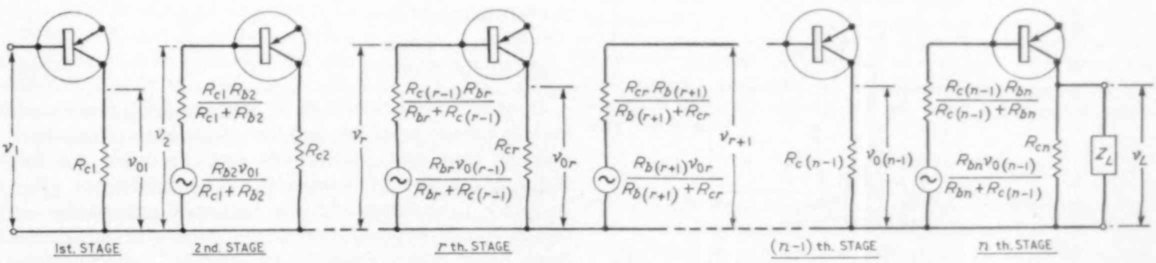

v_1
R_{c1}
v_{01}
v_2
$\frac{R_{c1}R_{b2}}{R_{c1}+R_{b2}}$
$\frac{R_{b2}v_{01}}{R_{c1}+R_{b2}}$
R_{c2}
v_r
$\frac{R_{c(r-1)}R_{br}}{R_{br}+R_{c(r-1)}}$
$\frac{R_{br}v_{0(r-1)}}{R_{br}+R_{c(r-1)}}$
R_{cr}
v_{0r}
$\frac{R_{cr}R_{b(r+1)}}{R_{b(r+1)}+R_{cr}}$
$\frac{R_{b(r+1)}v_{0r}}{R_{b(r+1)}+R_{cr}}$
v_{r+1}
$R_{c(n-1)}$
$v_{0(n-1)}$
$\frac{R_{c(n-1)}R_{bn}}{R_{c(n-1)}+R_{bn}}$
$\frac{R_{bn}v_{0(n-1)}}{R_{bn}+R_{c(n-1)}}$
R_{cn}
Z_L
v_L
1st. STAGE
2nd. STAGE
r th. STAGE
$(n-1)$ th. STAGE
n th. STAGE

Fig. 13. *Equivalent circuit for n-stage amplifier*

voltage gain and bandwidth of an amplifier having any number of cascaded stages may be obtained from values of transistor parameters and circuit components.

Overall Zero-Frequency Voltage Gain and Bandwidth of an *n*-Stage Amplifier

It follows from the equivalent circuit developed for one cascaded pair of similar stages, shown in Fig. 11, that the equivalent circuit for an *n*-stage amplifier having similar cascaded stages is as shown in Fig. 13. The final stage is assumed to be connected to a load impedance Z_L and the voltage gain of the amplifier will be the ratio of the voltage across this load, v_L to the input voltage v_1. The ratio $\frac{v_{0r}}{v_{0(r-1)}}$ for the *r*th stage will be given by Equ. (17) and is m', while the ratio v_{01}/v_1 for the first stage will be given by Equ. (3) and is m.

The overall voltage gain, m_T say, may then be found from the following formula.

$$m_T = \frac{(m')^{n-1} m}{1 + \frac{R_{cn}}{Z_L}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

It is observed that the input impedance of a stage in the amplifier depends on its position in the amplifier. For example, the input impedance of the *n*th stage will be that of a stage having an impedance Z_L connected across the collector resistance and will differ from the input impedance of a stage remote from Z_L. It is concluded that, although the ratios of actual voltages v_{r+1}/v_r may differ from stage to stage along the amplifier and be different from the ratio of the Thévenin generator voltages $v_{0r}/v_{0(r-1)}$, Equ. (20) is a precise formula for the overall voltage gain.

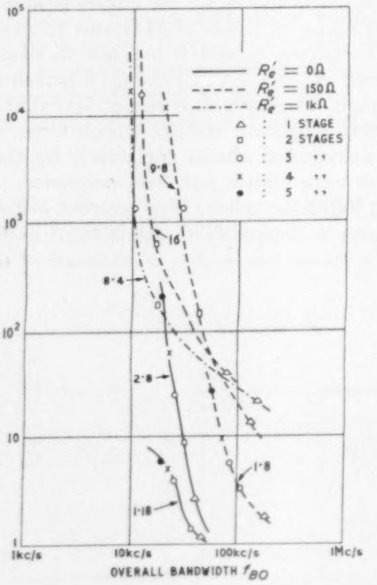


Fig. 14. *Overall zero-frequency voltage gain and bandwidth of amplifier having up to five stages. (Curves labelled with mean value of m_0')*

TABLE D

<table>
<tr><th rowspan="3">Amplifier Number</th><th colspan="5">Zero-Frequency Voltage gain of isolated stage</th><th rowspan="3">Overall Zero-Frequency Voltage Gain
m_{T0} (dB)</th><th rowspan="3">Overall Bandwidth
f_{B0} (kc/s)</th></tr>
<tr><th colspan="5">Stage Number</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>1</td><td>3·12</td><td>3·62</td><td>3·49</td><td>3·30</td><td>3·85</td><td>54·0</td><td>41</td></tr>
<tr><td>2</td><td>3·01</td><td>3·61</td><td>3·50</td><td>3·42</td><td>3·85</td><td>54·0</td><td>43</td></tr>
<tr><td>3</td><td></td><td></td><td></td><td></td><td></td><td>54·0</td><td>38</td></tr>
<tr><td>4</td><td></td><td></td><td></td><td></td><td></td><td>53·2</td><td>29</td></tr>
<tr><td>5</td><td></td><td></td><td></td><td></td><td></td><td>54·9</td><td>42</td></tr>
<tr><td colspan="6">Average Values</td><td>54·0</td><td>39</td></tr>
<tr><td rowspan="3">Theoretical Values</td><td colspan="2">Isolated Stage</td><td colspan="2">Cascaded Stage</td><td rowspan="3"></td><td rowspan="3">56·3</td><td rowspan="3">35·7</td></tr>
<tr><td>m_0</td><td>f_B (kc/s)</td><td>m_0'</td><td>f_B' (kc/s)</td></tr>
<tr><td>3.83</td><td>158.5</td><td>3.61</td><td>73.3</td></tr>
</table>

The overall zero-frequency voltage gain and bandwidth may be found from Equs (5), (6), (17) and (20). If R_L is the zero frequency value of Z_L, the overall zero frequency voltage gain will be given by

$$m_{T0} = \frac{(m_0')^{n-1} m_0}{1 + \frac{R_{cn}}{R_L}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

The overall bandwidth f_{B0} will be given by the condition

$$\left[1 + \left(\frac{f_{B0}}{f_B'}\right)^2\right]^{n-1} \left[1 + \left(\frac{f_{B0}}{f_B}\right)^2\right] = 2$$

The solution of a formula of this type is a familiar one in valve-amplifier theory and is given to a good approximation by

$$\left.\begin{aligned} f_{B0} &= \frac{1}{\sqrt{\frac{1}{f_x^2} + \frac{1}{f_y^2}}} \\ \text{where } f_x &= f_B' \sqrt{2^{\frac{1}{n-1}} - 1} \simeq \frac{f_B'}{\sqrt{n-1}} \\ \text{and} \quad f_y &= f_B. \end{aligned}\right\} \quad \ldots \quad (22)$$

When n is large, an approximate formula for the overall bandwidth is

$$f_{B0} = f_B' \sqrt{2^{\frac{1}{n}} - 1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

In order to test the formula, measurements were made on a five-stage amplifier and the values of zero-frequency voltage gain and bandwidth were recorded for five different batches of transistors. The results are given in Table D together with the corresponding theoretical values found by substituting the transistor parameters from Table B into Equs (5), (6) and (17) and applying Equs (20) and (22). The mean values of zero-frequency voltage gain and bandwidth found by experiment are 54 dB and 39 kc/s which compare favourably with the calculated values of 56·3 dB and 35·7 kc/s respectively. A comprehensive set of measurements was then made of the values of zero-frequency voltage gain and bandwidth of amplifiers comprising one, two, three, four and five similar stages, for each of three values of emitter resistance, with the final stage having a collector load resistance of R_c only. The combined results are plotted in Fig. 14. The values of externally added emitter resistor chosen represent a relatively large one of 1 kΩ, zero and an intermediate value of 150 Ω. The results may be compared with those for the isolated stage, Fig. 5, and it is observed that for zero-frequency voltage gains up to about 80 (38 dB) a single stage provides the maximum voltage gain-bandwidth product. The curves also show that a given voltage gain-bandwidth product may be produced by a minimum number of stages having an optimum value of emitter resistance.

If the cascaded stages in an amplifier are all different, the overall voltage gain m_{T0} and bandwidth f_{B0} may be determined by calculating the individual values of m_0' and f_B' from Equ. (17). By analogy with Equs (21) and (22), if m_{0r}' and f_{Br}' represent the values of m_0' and f_B' for the rth stage,

$$m_{T0} = \frac{m_{0n}' \, m_0'{}_{(n-1)} \, \ldots\ldots \, m_{02}' \, m_{01}}{1 + \dfrac{R_{cn}}{Z_L}}$$

and

$$f_{B0} \simeq \sqrt{\frac{1}{\dfrac{1}{f_{Bn}'^2} + \dfrac{1}{f_{B(n-)1}'^2} + \ldots\ldots + \dfrac{1}{f_{B2}'^2} + \dfrac{1}{f_{B1}^2}}}$$

Conclusions

It has been shown that a theory, based on a simple equivalent circuit for a transistor and involving approximations which reduce the voltage-gain expression to a form involving only one time-constant, is adequate to provide the variation of the amplitude of voltage gain of a common-emitter amplifier with frequency in a range up to 800 kc/s. It is shown that there is a maximum bandwidth for each value of emitter feedback resistance. Also, it was found that the bandwidth f_B is related by a simple reciprocal law to frequencies $f_{B\alpha}$ and f_{Bc} depending respectively on the bandwidth of the current gain α and the product of collector capacitance and base resistance. There is an optimum value of emitter resistance required to provide maximum zero-frequency gain-bandwidth product at a specified value of zero-frequency voltage gain which may be found from Equ. 14.

When a choice has to be made from several transistors of different types to provide the highest gain-bandwidth product, and if the transistor parameters are known, Equ. (15) may be applied.

The assumption that the output impedance is much larger than the collector resistance enables the theory for the isolated stage to be applied to cascaded stages. The relationship between zero-frequency voltage gain and bandwidth of the cascaded stage is found theoretically and good agreement with experimental results is observed up to the highest frequency used, 300 kc/s.

APPENDIX 1

Derivation of a Formula for the Voltage Gain as a Function of Frequency

The complete alternating-current equivalent circuit of the amplifier follows from Figs. 1 and 2.

The expression for the voltage gain m may then be derived as follows.

$$v_1 = -r_b \, . \, i_b - (R_e' + r_e) i_e$$

$$v_2 = -R_c \, i_b + R_c \, i_e$$

and

$$0 = -(R_c + Z_c) \, i_b + [(R_e' + r_e) + (R_c + Z_c) - \alpha Z_c] \, i_e$$

where Z_c is the impedance of C_c and r_c connected in parallel. Writing $R_e = R_e' + r_e$,

$$m = \frac{v_2}{v_1} = \frac{(R_e - \alpha Z_c) R_c}{(R_e + r_b)(R_c + Z_c) + r_b(R_e - \alpha Z_c)}$$

This equation may be rearranged in the form of complex numbers and it is convenient to use the quantities defined in Equ. (2) as follows

$$m = \frac{-a\{\alpha_0 - b[c - xy^2 + jy(c + x)]\}}{1 - \alpha_0 + b + d\,(c - xy^2) + jy\,[1 + b + d\,(c + x)]} \quad \ldots (24)$$

It is shown in Appendix 2 that the second term in the numerator of this expression may be neglected. In Appendix 3 it is shown that the frequency-dependent term $-dxy^2$ in the real part of the denominator may be neglected. When these terms are neglected, Equ. (24) reduces to Equ. (3) and takes the form of a quantity affected by a single time-constant; thus, if m_0 is the zero-frequency value of m and the magnitude of m is 3 dB below its zero-frequency value at a frequency f_B,
one may write

$$m = \frac{m_0}{1 + j\dfrac{f}{f_B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The quantities m_0 and f_B are the zero-frequency voltage gain and the bandwidth of the amplifier respectively,

Hence,

$$m_0 = \frac{-a \, \alpha_0}{1 - \alpha_0 + b + cd} \quad \ldots\ldots\ldots\ldots\ldots (26)$$

and

$$f_B = f_\alpha \frac{1 - \alpha_0 + b + cd}{1 + b + d\,(c + x)} \quad \ldots\ldots\ldots\ldots (27)$$

It is noted that if the collector load resistance R_c is increased, the values of a and d increase. Equation (26) indicates that m_0 approaches a limit

$$m_0 \rightarrow \frac{\alpha_0}{c\,(1 + b)} = \frac{\alpha_0 \, r_c}{r_b + R_e} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

It is misleading to put $R_c \rightarrow \infty$ in the formula for f_B above since the frequency-dependent terms discussed in Appendix 3 should then be included. This problem does not arise with m_0 for the frequency is then zero.

APPENDIX 2

Magnitude of the Terms in the Numerator of the Expression for Voltage Gain

The expression for voltage gain m is Equ. (24). The significant part of the numerator is the expression

$$1 - \frac{b}{\alpha_0} [c - xy^2 + jy\,(c + x)]$$

The frequency-dependent terms will have their largest values when the highest frequencies f and the highest values of R_e are used. Taking a frequency of 600 kc/s, the highest used, and a large value of R_e of 1 kΩ and substituting other values obtained from Table A, the above expression has a value $1 \cdot 17 \; \angle -8^\circ 37'$. Thus taking the most extreme conditions, if the second term is neglected the amplitude of the numerator is 17% too small and the phase angle is 8½° too large.

APPENDIX 3

The Magnitude of the Frequency-Dependent Term in the Real Part of the Denominator of the Expression for Voltage Gain

The expression for the real part of the denominator of the voltage-gain expression, Equ. (24), is

$$1 - \alpha_0 + b + d\,(c - xy^2)$$

If the frequency-dependent term is to be negligible, the following condition must apply,

$$f^2 \ll \frac{f_\alpha^2}{xd}(1 - \alpha_0 + b + cd)$$

The frequency at which the amplifier may be operated with useful voltage gain depends on the choice of values of R_e and R_c. It is shown earlier (The Relationship between Zero-Frequency Voltage Gain and Bandwidth) that for each value of R_e there exists a maximum value of bandwidth f_m', given sufficiently accurately by Equ. (13). The inequality becomes

$$f^2 \ll \frac{f_\alpha^2}{xd}\left[cd + \frac{f_m'}{f_\alpha}(1 + b)\right]$$

Conditions which make the right-hand side of this expression small will occur when f_m' is small and R_c large. It is observed from Fig. 5 that a small value of f_m' implies a small value of R_e. Two extreme conditions are considered:

(1) $f_m' = 48{\cdot}6$ kc/s, $R_e = 24\ \Omega$: With an emitter resistance of 24 Ω a large value of R_c would be 1 kΩ.

Then the inequality becomes, with f in kc/s,

$$f \ll \sqrt{196{,}600} = 443 \text{ kc/s}$$

Since the bandwidth of the amplifier with the values stated is approximately 40 kc/s, the condition that the frequency-dependent term is negligible is satisfied.

(2) $f_m' = 512$ kc/s, $R_e = 1{,}024\ \Omega$: With an emitter resistance of 1,024 Ω, a large value of R_c would be 10 k Ω.

Then the inequality becomes, with f in kc/s,

$$f \ll \sqrt{202{,}700} = 450 \text{ kc/s.}$$

Since the bandwidth of the amplifier with the values stated is 175 kc/s (with $m_0 = 9{\cdot}11$), the condition that the frequency-dependent term is negligible is satisfied.

It is noted that the tangent of the phase angle of the denominator of Equ. (24) is given by

$$y\frac{1 + b + d(c + x)}{1 - \alpha_0 + b + d(c - xy^2)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

It is clear that at high frequencies, the denominator of Equ. (29) may become small and, although its contribution to the modulus of the voltage gain would not then be significant, the value of the phase angle may be appreciably affected.

APPENDIX 4

Derivation of a Formula Relating the Zero-Frequency Voltage Gain to the Bandwidth

The variable R_e which is included in x and d is eliminated between Equs (5) and (6). The modulus of the zero-frequency gain m_0 is used so that the negative sign in the numerator of Equ. (5), which accounts for the phase reversal of a signal on passing through the amplifier, may be neglected. Then, making no approximations and writing

$$f_m = f_\alpha \frac{1 - \alpha_0 + b(1 + c)}{1 + b(1 + c + x)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

Equ. (6) becomes

$$f_B = \frac{\alpha_0 f_m}{\alpha_0 + m_0(1 + b)\left[\frac{f_m}{f_\alpha}(c + x) - c\right]} \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$m_0 \rightarrow \infty$

If $m_0 \rightarrow \infty$, it follows from Equ. (31) that

$$m_0 f_B = \frac{\alpha_0}{(1 + b)\left[\frac{1}{f_c} + c\left(\frac{1}{f_\alpha} - \frac{1}{f_m}\right)\right]} \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

The term in f_α is first considered using the transistor parameters of Table A. The value of cf_c is 7,958 c/s and the value of f_α is nearly 1 Mc/s so the term in f_α may be neglected.

The term in f_m is largest when R_e is smallest; i.e., $R_e = r_e$. But applying the more stringent condition, $R_e = 0$, $b = 0$ and $f_m \rightarrow f_\alpha(1 - \alpha_0) \simeq 20$ kc/s.

It is clear that transistors having particularly low values of f_α when used with very small values of R_e will not fulfil the condition $cf_c \gg f_m$ but, in general, the Equ. (32) may be written

$$f_B \rightarrow \frac{\alpha_0 f_c}{(1 + b)\, m_0} = \frac{\alpha_0}{2\pi C_c\, m_0 (r_b + R_e)} = f_{BC} \text{ say.} \quad \ldots \quad (10)$$

This equation is shown in the earlier section (The Relationship between Zero-Frequency Voltage Gain and Bandwidth) to provide accurately the asymptotes of the curves in Fig. 6 for all values of R_e employed. If the value of f_α tends to infinity then Equ. (10) may be regarded as the formula relating high values of m_0 to f_B when the reactance of the collector capacitance is the only frequency-dependent parameter in the circuit.

$f_c \rightarrow \infty$

When $f_c \rightarrow \infty$ (i.e., $x \rightarrow 0$) Equs (30) and (31) provide the following equation for f_B, which is then called $f_{B\infty}$.

$$f_{B\infty} = f_\alpha \frac{1 - \alpha_0 + b}{(1 + b)(1 - cm_0)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

APPENDIX 5

Optimum Value of Emitter Resistance for Maximum Bandwidth at a Given Zero-Frequency Voltage Gain

The expressions for $f_{B\infty}$, Equ. (33), and f_{BC}, Equ. (10), are substituted into Equ. (13) and the following equation for f_B is obtained,

$$f_B = \frac{\alpha_0 f_\alpha (1 - \alpha_0 + b)}{(1 + b)\left[\alpha_0(1 - cm_0) + m_0 x(1 - \alpha_0 + b)\right]} \quad \ldots \quad \ldots \quad (34)$$

Differentiating the expression for f_B with respect to R_e and equating to zero results in the following equation,

$$\alpha_0^2(1 - cm_0) = m_0 x(1 - \alpha_0 + b)^2 \text{ .}$$

From this equation a formula for R_e in terms of transistor parameters and m_0 is obtained

$$R_e = \alpha_0 r_b \sqrt{\frac{1 - cm_0}{m_0 x}} - r_b(1 - \alpha_0) = R_{eo}, \text{ say} \quad \ldots \quad \ldots \quad (35)$$

Equation (35) gives the value of R_e which makes f_B a maximum or minimum, but clearly it is the maximum value which it yields.

If the term containing r_e is neglected and m_0 is less than 50, the value found for R_{eo} is less than 5% too large. If the term $r_b(1 - \alpha_0)$ is neglected and m_0 is less than 50, the value for R_{eo} is less than 7% too large. Also, if the factor α_0 multiplying the first term on the right-hand side of Equ. (35) is made equal to unity, the value found for R_{eo} is 2% too large. Thus, if m_0 is less than 50, a value of R_{eo} which is less than 14% too large may be found from the formula,

$$R_{eo} = \frac{r_b}{\sqrt{m_0 x}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

When m_0 is less than 10, Equ. (36) provides a value of R_{eo} which is less than 6% too large.

Using Equ. (2), Equ. (36) becomes Equ. (18).

APPENDIX 6

The Maximum Voltage Gain-Bandwidth Product

In Appendix 5 the formula in terms of transistor parameters was found for the emitter resistance R_{eo} required to make the bandwidth a maximum for a given value of zero-frequency voltage gain. This maximum bandwidth will be called f_{Bo} and now the object is to obtain an equation for f_{Bo} in terms of transistor parameters and m_0. Since Equ. (34) gives f_B in terms of R_e, transistor parameters and m_0 only, the desired result may be obtained by combining Equ. (34) with the formula for R_{eo}, as follows.

When the emitter resistance has its optimum value R_{eo}, it has been shown in Appendix 5 that the term $-r_b(1 - \alpha_0)$ in Equ. (35) may be neglected and Equ. (35) reduces to Equ. (36). If R_{eo}, given by Equ. (36), is substituted into Equ. (34), omission of the terms in $(1 - \alpha_0)$ on the right-hand side of Equ. (34) will actually cause an improvement in accuracy. Thus,

$$\frac{1}{f_{Bo}} = (1 + b)\left[\frac{1 - cm_0}{bf_\alpha} + \frac{m_0}{\alpha_0 f_c}\right]$$

Substituting for b from Equ. (36),

$$\frac{1}{f_{Bo}} = \frac{1}{f_\alpha}\left[(1 - cm_0)(1 + \sqrt{m_0 x}) + \frac{1}{\alpha_0}\sqrt{m_0 x}\right] + \frac{m_0}{\alpha_0 f_c}$$

The last term on the right-hand side is the value of $1/f_{BC}$ when R_e is zero.

Since α_0 has a value of 0·98,

$1 + (1/\alpha_0) \simeq 2.$

Therefore, to a good approximation,

$$\frac{1}{f_{Bn}} = \frac{1}{f_a}[1 - \epsilon m_n + (2 - \epsilon m_n)\sqrt{m_n \epsilon}] + \frac{m_n}{\kappa_n f_c} \qquad \cdots \qquad \cdots \quad (37)$$

Equation (37) gives the maximum bandwidth f_B obtainable at a given zero-frequency voltage gain m_n in terms of transistor parameters and m_n. When ϵ is made zero, Equ. (37) reduces to the form of Equ. (15).

APPENDIX 7

The Output Impedance of a Common-Emitter Amplifier

Referring to the equivalent circuit of the transistor, Fig. 1, and considering the base and emitter terminals to be short-circuited, if v_k is the voltage of the collector terminal with respect to the emitter terminal,

$$v_k = Z_c i_b - [r_e + (1 - \alpha) Z_c] i_e$$

and

$$0 = r_b i_b + r_e i_e$$

If there is a resistance R_e' in the emitter circuit as shown in Fig. 2 then r_e must be replaced by R_e in the equations and, applying Equ. (2), the formula for the output impedance Z_o is obtained.

$$Z_o = \frac{v_k}{i_b - i_e} = \frac{R_e}{1 + b} + \frac{Z_c}{1 + b}(1 - \alpha + b) \qquad \cdots \qquad \cdots \quad (38)$$

APPENDIX 8

Collector Load Resistance Required to Obtain the Maximum Value of the Cascaded Voltage Gain

On differentiating with respect to R_c the expression for m_n', obtained from Equ. (17) with $y = 0$, the value of R_c which makes m_n' a maximum, R_{cn} say, is found. Writing $R_b/r_b = \epsilon$ and neglecting ϵ compared with unity,

$$R_{cn} = R_e \frac{\epsilon}{b}\left[\frac{\epsilon(\alpha_0 + \alpha - 1 - \epsilon b)}{1 - \alpha_0 + b} - 1\right]^{\frac{1}{2}} \qquad \cdots \qquad \cdots \qquad \cdots \quad (39)$$

If R_b has a large value compared with R_e, the value of R_{cn} may be found from the above formula, putting $\epsilon \to \infty$, and Equ. (19) is obtained.

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DIFFERENTIAL POLAROGRAPHIC ANALYSER

A new instrument which can be used as an analytical tool for chemical and metallurgical examinations of many kinds has recently been produced by Nash & Thompson. Among particular applications are soil analysis, studies of the oxygen content of blood plasma; the investigation of respiration or photosynthesis in living cells; determination of the inorganic constituents of water, etc.

When used as a means of detecting impurities in foodstuffs, the differential polarographic analyser (D.P.A.) can identify traces of metals at concentrations as low as 0·02 parts per million and can carry out quantitative analyses to an accuracy of ±1% to ±2% in solutions containing 1 part per million of the species being investigated.

Polarography depends on the measurement of small electric currents which are caused to flow by the ionization of different atoms or molecules under the influence of a steadily increasing p.d. between two electrodes in a solution.

The atoms and molecules of every oxidizable or reducible substance have different 'ionizing potentials' in an electrolyte; they therefore give rise to different currents for given voltages. In the D.P.A., these currents are displayed on a c.r.t. in the form of current—voltage curves, from which the various substances and their concentrations in the supporting electrolyte can be identified.

The use of two cells in differential polarography simplifies the work because by having two identical input circuits in the display equipment the currents through the cells can be applied to the input differential amplifier in opposition. This means that when the two cells contain solution of identical chemical composition, the currents passing through them cancel out and the display on the c.r.t. becomes a straight line. Thereafter, the introduction into either cell of a different chemical element will produce a waveform which is characteristic of that new element. In this way, the analytical process is greatly simplified.

The currents through the two cells are derived from a d.c. ramp potential which is applied to the cells. The system comprises a timing circuit, sweep generator and amplifier, oscillator, two modulators, differential amplifier, phase-sensitive detector, and the normal c.r.o. X and Y amplifiers.

In operation, the sweep generator develops the d.c. ramp voltage which, through the sweep amplifier, is fed to the two polarographic cells. The resultant currents are then fed into two modulators, one for each cell, which are excited by an oscillator, and the outputs from the modulators are connected to the differential amplifier. The output from the differential amplifier, consisting of the sum or difference of the inputs, is fed to the main Y amplifier and rectified by the phase-sensitive detector before it is connected to the Y deflection system. The output from the sweep generator is fed directly to the X-deflection amplifier.

Polarograph analyser connected to a pair of cells

THOMSON LECTURE

The Council of The Society of Instrument Technology have decided to inaugurate an Annual Lecture on a general subject. The first President of the Society, Sir George Thomson, M.A., F.R.S., has not only consented to give his name to this lecture but also to deliver the first one, which is entitled "The Installation of Science".

This will be at 6 p.m. on Thursday 19th October 1961, at The Royal Institution, 21 Albemarle Street, W.1. Admission will be by ticket only, applications for which should be made to the Secretary, The Society of Instrument Technology, 20 Queen Anne Street, London, W.1.

NEW GERMAN CYBERNETICS JOURNAL

Springer-Verlag (Heidelberger Platz 3, Berlin-Wilmersdorf, Germany) have recently published a new journal *Kybernetik*. With the exception of one article, the first issue was printed in German; future issues will include contributions in German and in English and will cost DM 12.80 each.

Bandwidth Compression of Speech

SAMPLING IN THE FREQUENCY DOMAIN

J. Das, M.Sc., A.M.I.E.E.*

SUMMARY. *A method of sampling in the frequency domain alone and consequent bandwidth reduction of time-varying signals are discussed here. The signal is sampled by bandpass filters with centre frequencies spaced uniformly and the different channels are multiplexed directly. In the receiver, the sampled signal is mixed with the sidebands generated in a balanced modulator using a low-frequency carrier and the samples themselves. The corrected signal is then found to have satisfactory intelligibility even with large bandwidth compression.*

Recently interest has been shown by various authors in sampling of speech signals both in the time-frequency domain and in the frequency domain alone. Peterson and Subrahmanyam[1] have attempted to compress the effective speech bandwidth by simultaneous sampling in the frequency and time domains, but the results are not very encouraging. On the other hand, Kryter[2] has shown that satisfactory communication could be obtained by sampling the speech with only three 500-c/s bandpass filters. But the total bandwidth in this case comes to about 2,340 c/s at 30-dB points and hence the bandwidth compression ratio is rather small. Moreover, due to the non-uniform sampling, the different bands have to be translated to form a compact spectrum so that this may be used with advantage in frequency-division multiplex systems. A method of uniform sampling in the frequency domain along with a necessary correction is proposed here to make direct multiplexing possible and to obtain a better bandwidth-compression ratio.

Theory of Sampling

A finite sample of a signal, even time-varying, may be represented as a sum (or integral) of exponentials as

$$F(\omega,t) = \Sigma A_n \exp(p_n t) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where, A and p are complex. The waveform, however, has to be completely known for all time. For any random function of time, such as speech or a continuous information-bearing process, a possible representation in the time-frequency plane is of the form[3]

$$F(\omega,t) = \Sigma A_{mn} U_{mn}(t) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

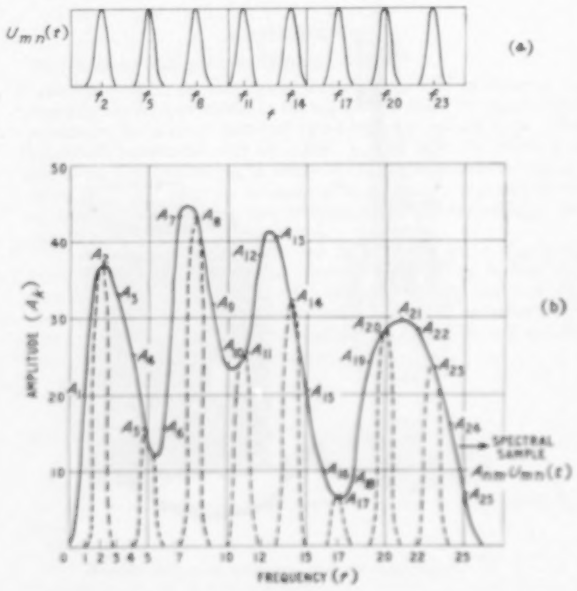
where, A_{mn}, given by $S(m\theta, n/\theta)$, are the coefficients of the sampling functions $U_{mn}(t)$ corresponding to the cross-points of any grid laid on the t-ω plane, for which the separation of the crosslines is θ in time and $1/\theta$ in frequency. It then follows that the signal may be represented by $(m \times n)$ numbers, once the choice of θ has been made, and the sampling may even be done either in the time domain or in the frequency domain alone. Ideally, a signal of bandwidth W and duration T requires $2TW$ numbers to specify it completely. But slowly-varying signals[4], such as speech, have much less essential information-contents than specified by $2TW$ numbers.

Moreover, Dudley, in his classical work[5], has shown that the voiced sound may be represented as

$$F_v(\omega,t) = s(t) \sum_{k=1}^{n} r(\omega,t)\, A_k \cos\left[kP \int_0^t p(t)dt + \theta_k\right] \qquad (3)$$

where the carrier is composed of n audible harmonics of relatively high frequencies having amplitude A_k, frequency kP and phase θ_k; $s(t)$ is the switching function, $p(t)$ the inflecting factor and $r(\omega, t)$ the effect of selective

Fig. 1 (a). *Nature of sampling functions;* (b). *a typical amplitude-frequency curve and its samples (dotted curves)*



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* *Indian Institute of Technology, Kharagpur, India.*

transmission. The three message functions $s(t)$, $p(t)$ and $r(\omega,t)$ produce the necessary modulation processes on the carrier at the relatively low syllabic rate. The total message information is mainly dependent on these slowly-varying parameters. The unvoiced sound is only a degenerate case of Equ. (3). Analysis with vocoders has shown that the message currents are generally restricted to a bandwidth of 50 c/s only.

Both the above representations tend to show that a good approximation of speech-like signals should be possible with $(m \times n)$ much less than $2TW$. Experiments have been initiated to determine a minimum $(m \times n)$ for speech signals and then to add a correction to the sampled signal to improve its intelligibility and quality. Surprisingly enough, it has been found that sampling in the frequency domain alone produces reasonably intelligible signals, even when the total useful bandwidth of the system has been reduced to 25% of the original bandwidth.

The effect of sampling in the frequency domain is shown in Fig. 1, where the instantaneous amplitude of the component frequencies may be represented by A_1, $A_2 \ldots . A_n$. The sampled signal, consisting of A_2, A_5, A_8, etc., has holes in the amplitude-frequency curve and consequently the intelligibility and naturalness deteriorates. However a process, similar to the pulse-lengthening generally used in time-division multiplex systems, helps to improve the quality of the signals. The holes created by sampling are now filled by inserting sidebands corresponding to the accepted bands, but differing in frequency by $(\Delta f/3)$ only, where $\Delta f = (f_5 - f_2) = (f_8 - f_5) = (f_n - f_{n-3})$. The reconstructed amplitude curve would then be of the staircase type and the new frequency-domain representation of the signal would be

$$F(\omega,t) = F_1(\omega,t) + F_2(\omega,t) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$$

where,

$$F_1(\omega,t) = A_{m2}U_{m2}(t) + A_{m5}.U_{m5}(t) + A_{m8}U_{m8}(t) + \quad (5)$$
$$= \text{the sampled signal.}$$

and

$$F_2(\omega, t) = A_{m2}[U_{m1}(t) + U_{m3}(t)] +$$
$$A_{m5}\,[U_{m4}(t) + U_{m6}(t)] + A_{m8}[U_{m7}(t) + U_{m9}(t)] + \quad (6)$$
$$= \text{the sidebands of the sampled signal.}$$

Fig. 2 shows this staircase amplitude curve and the spectrum is now seen to be filled up. Some measure of the error in the synthesized signal may be obtained from the relations[6]

$$\text{Variation} = \int |D'(f)|\,df \qquad \ldots \qquad \ldots \qquad (7)$$
$$\text{Difference} = \int |D(f)|\,df \qquad \ldots \qquad \ldots \qquad (8)$$
$$\text{Square of difference} = \int |D(f)|^2\,df \qquad \ldots \qquad \ldots \qquad (9)$$

where, $D(f)$ is the difference between the original spectra and the synthesized spectra, the average difference over all f being normalized to zero.

Experimental Results

In a practical sampling system, the coefficient A_{mn} will represent a small band of frequencies and it is necessary to determine the minimum number of bands as well as the optimum width of each band that are to be transmitted for reconstructing an intelligible signal. The experimental set-up for the purpose is shown in Fig. 3. In the transmitter, the different bands are selected by bandpass filters only and, in the receiver, the transmitted signal as well as the sidebands generated in the balanced modulator are added together before final amplification. To avoid disturbing transients in the output, the sampling filters should have smooth cut-off—somewhat gaussian, even at the expense of more effective bandwidth of the system. Leakage of the carrier in the receiver causes masking of the signal and a sharp bandstop filter is used to eliminate it from the output. Alternatively, the speech signal may first be translated to a higher band of frequencies, say 6–10 kc/s, then sampled, mixed with its sidebands and retranslated to the original frequency band.

Preliminary experiments with six filters, each having

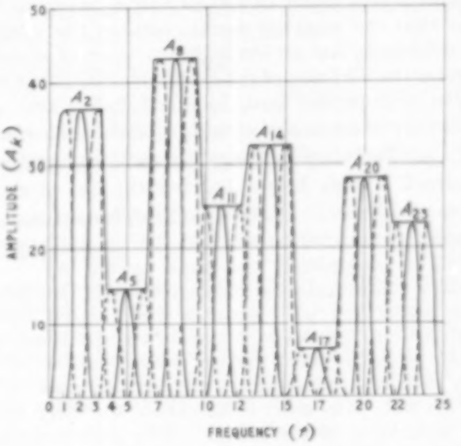
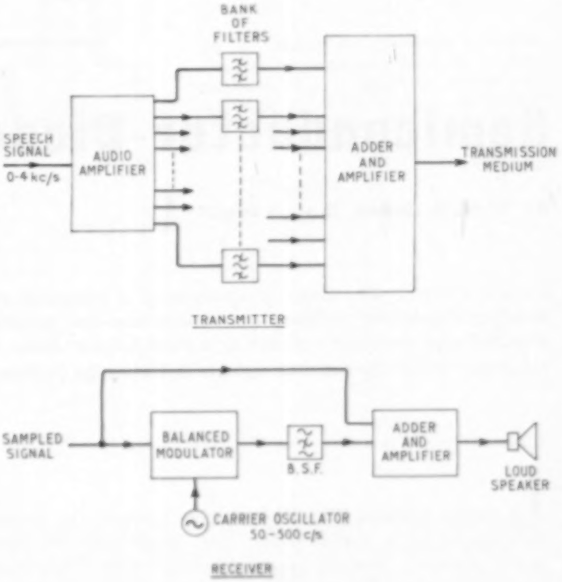


Fig. 2. *The staircase type of the reconstructed amplitude-frequency curve. The samples and the corresponding sidebands (dotted) are shown*



TRANSMITTER

RECEIVER

Fig. 3. *Experimental set-up for sampling and reconstruction of the signal*

a nominal bandwidth of 200 c/s and Δf equal to 600 c/s, show that the received signal, without the addition of the sidebands, has an intelligibility score of about 85%, whereas the addition of the sidebands makes the receiver output near-perfect both in intelligibility and quality. With only three samples, each of 200-c/s bandwidth, a 90% intelligibility in the corrected receiver output is obtained. With larger bandwidth, the intelligibility scores are better. However, the optimum shift of the samples is found to be about 150–250 c/s, and any attempt to fill up completely the gaps having $\Delta f > 750$ c/s tends to deteriorate the receiver output. Initially, tests were performed with time-varying carrier fed to the modulator to cover wider gaps in the spectrum, but the results with the fixed carrier were found to be better. The sampling bandwidth may be decreased to about 100 c/s but, with lesser bandwidths, the filter transients are prominent in the output. The carrier suppression in the modulator has to be better than 50 dB. Because of the smaller effective bandwidth of the system, it is expected that the signal-to-noise ratio in the output will also improve. The proposed technique is being studied in detail at present and further results will be communicated shortly. It may, however, be concluded that in the system proposed above, since the sampling is done at equal intervals in the frequency scale, different channels may be multiplexed directly without any frequency-translation of the different filter outputs. The bandwidth compression possible here will also be more than that obtained so far by other similar methods.

Acknowledgement

The author records his thanks to Mr. S. K. Mullick for illuminating discussions.

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Semiconductor-Diode Waveguide Switch

By T. H. B. Baker, B.A., A.Inst.P.*

SUMMARY. *The theory of operation of a waveguide switch is given, relating the switching ratio, bandwidth and power-handling capabilities to diode and transmission-line parameters. The construction and performance of practical devices are described with particular reference to a p-i-n junction diode. The performance is illustrated by the results of tests made at high r.f. power levels, the transient response and the noise performance.*

In many systems it is desirable to control the power flowing along a transmission line, preferably by electronic means. Several types of device exist[1], but each has some particular disadvantages. Mechanical devices are slow in operation; discharge tubes introduce noise; ferrite devices are relatively slow and need large control power; and point-contact diode devices only handle a few milliwatts of power.

This article describes a waveguide switch using a junction diode[2] and designed for frequencies around 10,000 Mc/s, although the usefulness is by no means confined to this region. The device is compact, has a good switching ratio over a broadband, handles moderate powers and has good switching speeds.

Circuit Theory

If we consider a simple transmission line shunted by an admittance Y it is clear that the power which can flow along the line is controlled by the magnitude of Y. In particular, if Y is zero, there is no loss of power, but if Y is infinite the energy flow is short-circuited. For the general case where $Y = G + jB$, the insertion loss is given by

$$L = (1 + Z_0 G/2)^2 + (Z_0 B/2)^2 \qquad .. \qquad .. \qquad (1)$$

(For simplicity, it is assumed that the internal impedance R_g of the generator and the load impedance R_l are each equal to the characteristic impedance Z_0 of the transmission line.)

It is shown in Ref. 2 that the p-i-n junction diode

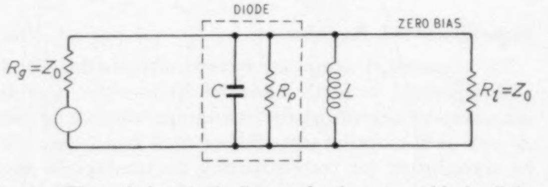

Fig. 1. *The equivalent circuit diagram for the system with the diode at zero bias*

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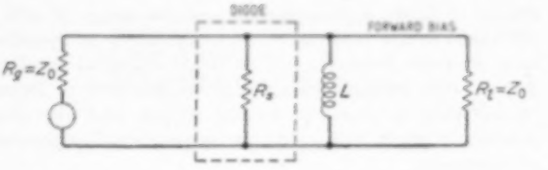


Fig. 2. *The equivalent circuit diagram for the system with the diode at forward bias*

behaves at microwave frequencies as a non-linear resistor whose value depends on the bias applied. At zero or negative bias the resistance (R_p) is high—of the order of several kilohms; but at positive bias the resistance (R_s) is low—only a few ohms. Fig. 1 shows the equivalent circuit of the system with the diode at zero bias and Fig. 2 that at forward bias ('Forward bias' usually means a voltage across the diode of 1·0 to 1·5 V, corresponding to a current of 20 to 100 mA).

The values of insertion loss L_0 at zero bias and the attenuation L_+ at forward bias are respectively

$$L_0 = (1 + Z_0/2R_p)^2 + (Z_0\,\omega\,C/2)^2 \quad .. \quad .. \quad (2)$$

and

$$L_+ = (1 + Z_0/2R_s)^2 \quad .. \quad .. \quad .. \quad .. \quad (3)$$

The insertion loss at zero bias may be reduced by placing an inductance L across the transmission line to tune out the small shunt capacitance C, at the centre of the frequency band, ω_0. The effect of this inductance upon the attenuation L_+ is usually negligible.

We therefore have

$$L_0 = (1 + Z_0/2\,R_p)^2 \quad .. \quad .. \quad .. \quad .. \quad (4)$$

$$L_+ = (1 + Z_0/2\,R_s)^2 \quad .. \quad .. \quad .. \quad .. \quad (5)$$

The values of L_0 and L_+ required determine the ratios of R_p and R_s to the transmission-line characteristic impedance Z_0. As an example, let us take $L_0 = 0{\cdot}5$ dB and $L_+ = 20$ dB, in a waveguide of impedance 500 Ω. Then we find that $R_s = 28\ \Omega$ and $R_p = 4{,}160\ \Omega$. We must note here that L and C only cancel out at the centre frequency ω_0; at any other frequency ω we must substitute the appropriate value of B in Equ. (1), where

$$B = (\omega\,C - 1/\omega\,L) \quad .. \quad .. \quad .. \quad .. \quad (6)$$

and

$$\omega_0\,C = 1/\omega_0\,L \quad .. \quad .. \quad .. \quad .. \quad .. \quad (7)$$

If ω is not very different from ω_0 we can write

$$B = C\,\Delta\,\omega \quad .. \quad .. \quad .. \quad .. \quad .. \quad (8)$$

where $\Delta\,\omega = 2\,(\omega_0 - \omega)$. Let us calculate the bandwidth $\Delta\,\omega$ within which the attenuation due to susceptance is less than 0·5 dB (a factor of 1·12).

Then at the limits of the band

$$1{\cdot}12 = (1 + Z_0\,B/2)^2$$

$$\text{or } 0{\cdot}12 = (Z_0\,B/2)^2 = (Z_0\,C\,\Delta\,\omega/2)^2$$

With $Z_0 = 500\ \Omega$ as before, and taking a typical value for the capacitance of 0·14 pF and a frequency of 9,375 Mc/s, we find that the total bandwidth (expressed as a percentage of ω_0) is 14%. The bandwidth depends on the characteristic impedance of the transmission line and the capacitance of the diode, and is greatest when these are both small.

Also of interest is the fraction of power absorbed by the diode. This is given by Uhlir[2] as

$$\frac{\text{Power dissipated in diode}}{\text{Power available}} = \frac{Z_0}{\text{insertion loss} \times R}$$

For the case described above, this turns out to be 11% at zero bias and 18% at forward bias.

Construction of Devices

Narrow-band System

In the narrow-band device the diode, which is encapsulated in a modified CV2355 silicon-crystal valve body, is mounted at the end of a short coaxial line coupled to the waveguide (Fig. 3). This is equivalent to placing the diode at the end of a half-wave stub shunted across a transmission line. An approximate equivalent circuit is given in Fig. 4. The system is inherently narrow-band owing to the change in electrical length of the stub with frequency. The variation of insertion loss to be expected with an ideal diode (i.e., one for which $R_p = \infty$ and R_s and $C = 0$) has been calculated and is shown in Fig. 5 together with curves for a real diode.

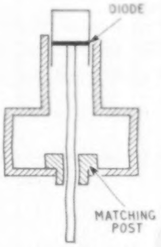


Fig. 3. (*Left*) *A narrow-band mount*

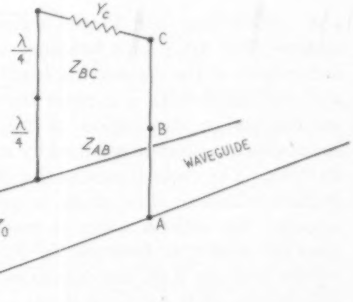


Fig. 4. (*Below*) *An approximate equivalent circuit for the narrow-band device.* Y_C = *diode admittance,* Y_A = *admittance measured in waveguide,* $Z_0 = 500\ \Omega$, $Z_{BC} = 66\ \Omega$, *and* $Z_{AB} = 96\ \Omega$

Typical performance figures are 1-dB insertion loss at zero bias and greater than 20-dB attenuation at forward bias. A small amount of the zero-bias loss may be attributed to losses in the holder and poor contacts. The insertion loss can also be improved slightly by applying a negative bias of about 50 V.

When setting up the unit, the position of diode and inner conductor in the waveguide may be adjusted to tune for minimum insertion loss. The dimensions of the matching post have been chosen to give maximum attenuation at forward bias with the above setting.

Broad-band System

In this system an attempt has been made to realize the simple circuit of Fig. 1 by tapering the waveguide

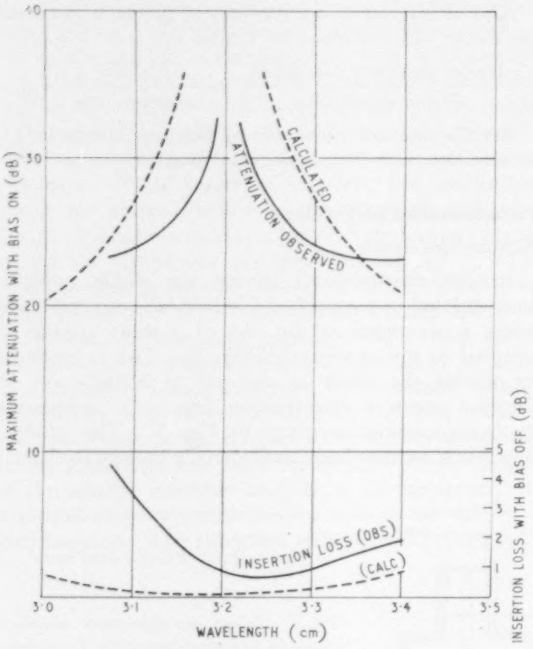


Fig. 5. *Performance of the door-knob holder with a typical diode; values calculated for an ideal diode*

and connecting the diode across the low-impedance section (Fig. 6). This has two advantages in that the inductance of the connecting leads is kept to a minimum, and the bandwidth is greater because the characteristic impedance in the region of the diode is low. Lead inductance is further reduced by making the lead conical in form. The capacitance of the diode is tuned out using inductive irises. The diode is not encapsulated in this system; the silicon wafer is mounted on the end of a plunger which is inserted so that the tip of the cone makes contact with the junction. The performance of this device with a typical diode is shown in Fig. 7.

Theory of P-I-N Diode

We now consider in more detail the action of the p-i-n structure at microwave frequencies. A diode is depicted in section in Fig. 8. The 'intrinsic' region is in fact a region of fairly high resistivity, of the order of 100 to 500 Ω/cm so the diode might more correctly be described as a p^+-n-n^+ structure. The usual material is silicon doped with phosphorus; the p^+-n junction is formed by alloying aluminium to the silicon and the n-n^+ junction is made with gold containing a small percentage of antimony.

We consider first the properties of the diode at zero bias. The first thing we notice is that the properties measured in the microwave region are quite different from those measured at lower frequencies. The quantities given in Table 1 are the measured capacitance and resistance of the equivalent parallel circuit for a typical diode.

TABLE 1

Frequency (Mc/s)	*Capacitance* (pF)	*Resistance* (k Ω)
5	1·5	20·0
9,375	0·2	2·0

The difference may be explained thus: associated with the p^+-n junction is the transition region or depletion-layer capacitance which is voltage dependent. This is shunted by the 'barrier' resistance which is very high. However, in series with the transition region is the intrinsic region, which may also be represented by a capacitance shunted by a resistance. The numerical values of the two capacitances are given roughly by the parallel-plate capacitor formula, taking the area as that of the junction, and the thickness as that of the transition region or the intrinsic region, whichever applies. It is found that the depletion-layer capacitance is about ten times the intrinsic-region capacitance. At low frequencies the impedance of the transition region is high compared with the resistance of the intrinsic region; therefore the quantities measured at 5 Mc/s are substantially the transition-region capacitance and resistance. However, at much higher frequencies (e.g., 9,375 Mc/s), the impedance of the transition-region capacitance is much lower than the impedance of the intrinsic region, so that measurement at 9,375 Mc/s yields essentially the capacitance and resistance of the intrinsic region. The equivalent circuit of Fig. 9 accounts quite well for the measurements quoted above. (The values are typical of those calculated from the dimensions of diodes.)

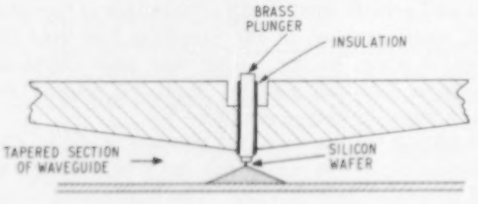
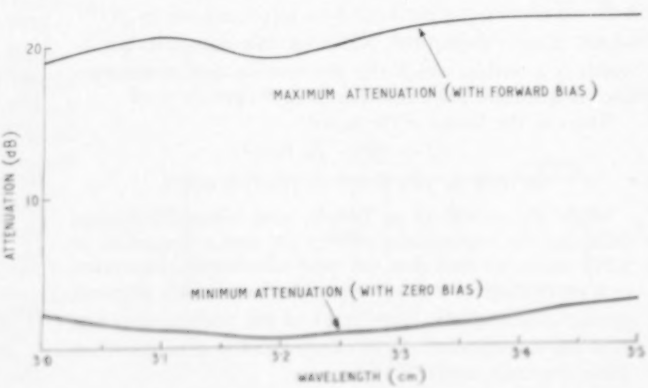


Fig. 6. (*Above*) *A broad-band mount*

Fig. 7. (*Right*) *The variation of attenuation with wavelength for a broad-band mount*



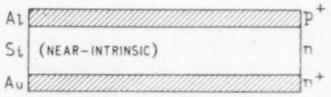


Fig. 8. *A schematic cross section of a p-i-n diode*

It was stated earlier that under forward-bias conditions the diode behaves as a low resistance. This is because the effective conductance is increased by the injected carriers (Ref. 2, Section VI, p. 1106). The conductance increases with the bias current.

Construction of Diode

Several methods have been used to make the junction. At first this was formed by alloying 5-mil aluminium wire to the silicon, but it was found that the extent of alloying could not be closely controlled. Another method, which gives more consistent results, is to evaporate aluminium on to the silicon surface and then alloy, the junction area being defined by ultrasonic cutting. A piece of silver wire is bonded to the aluminium to give a good contact. A simpler method is to evaporate a small dot of aluminium through a mask, followed by alloying; a silver wire is then bonded on as before.

The silicon slice is gold-plated on the lower surface and is alloyed to a fernico base using gold foil containing a small percentage of antimony. The fernico base is soldered to a shortened CV2355-type body, the wire being fed through the pin.

High-Power Switching

The low-power-level switching performance has already been described in an earlier section (Construction of Devices). One of the features of the junction diode switch is its ability to handle comparatively high c.w. and pulse powers and its performance in this field will now be described.

C.W. Measurements

C.W. power of up to 10 W was obtained from a SZ21 klystron followed by a ferrite isolator to protect the klystron from load variations. A directive coupler (with crystal detector) on each side of the switch unit enabled the incident and transmitted powers to be compared. The line was terminated in a high-power matched load. At power levels up to 5 W the switch behaved as in the low-level tests but, at higher incident powers, the device began to attenuate without the application of external bias. Only a small part of this effect could be attributed to self-bias because the diode is a very poor rectifier at microwave frequencies and the rectified current at 10 W is only about 50 μA. Furthermore, the effect was still present when the diode was open-circuited. The switch remained in the attenuating condition until the power level was reduced, when it returned quite suddenly to its transmitting state. The temperature of the diode body near the silicon was measured with a thermocouple and it was found that the anomalous attenuating condition was accompanied by a sharp rise in temperature.

The explanation of the phenomenon is believed to be as follows: as the power absorbed by the diode increases, its temperature rises and the resistivity of the silicon decreases. This alters the impedance of the diode so that it is better matched to the coaxial line and absorbs more power. The effect is cumulative and only ceases when further absorption of power would begin to mismatch the diode again and so reduce the power absorbed. The diode remains in this condition until the temperature is reduced, either by reducing the input power or by cooling the diode externally. If left too long in the heated condition, permanent damage to the diode results.

Power levels of up to 10 W can be handled provided the diode is not left for more than a few seconds in the matched condition. (This usually occurs for forward-bias currents in the region of 2 mA. It was shown in an earlier section (Circuit Theory) that only a fraction of the waveguide power is actually absorbed in the diode when it is in the 'on' or the 'off' condition.)

Pulse Measurements

Pulse measurements were made with an apparatus similar to that used for c.w. measurements except that the working section of waveguide was coupled through a 16-dB directive feeder to a magnetron delivering 40 kW peak power with a pulse length of 1 μs and a repetition rate of 1000 p.p.s. The maximum peak power in the working section was thus 1 kW, and this could be reduced using a variable attenuator, the actual power level being determined with a thermistor bridge. The transmitted power was rectified and displayed on an oscilloscope after amplification. The transmitted power was measured with a calibrated variable attenuator in the detector arm, which was used to bring the oscilloscope trace to a pre-determined level.

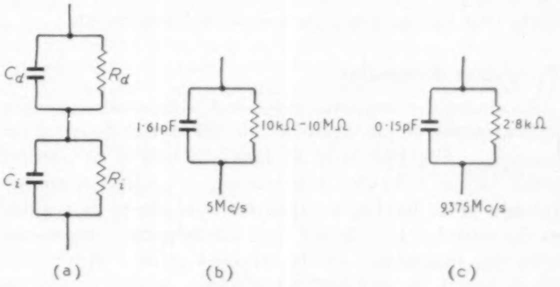


Fig. 9. *The basic equivalent circuit of the transition region or depletion layer and intrinsic region of a p-i-n diode (a). At 5 Mc/s, the transition region predominates (b) while at 9,375 Mc/s the intrinsic region is the most effective; values are typical of those calculated from the dimensions of diodes*

For pulse powers greater than about 60 W the transmitted pulse was distorted, the insertion loss apparently increasing during the pulse. As the peak power was increased the average insertion loss also increased. However, the attenuation varied very little (Fig. 10). In this case, it is possible that the increased minimum attenuation is due to rectified current causing self-biasing.

The fact that the minimum attenuation increases with power level and that the pulse is distorted is not neces-

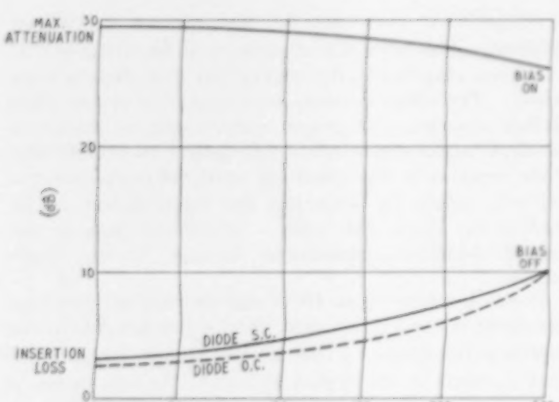


Fig. 10. *Variation of loss and attenuation with r.f. power level*

sarily a disadvantage; for example, in the case where one is protecting a point-contact diode, and is concerned with the maximum attenuation, which has been shown to vary little with power level.

Some diodes still functioned at 1-kW peak power, but others failed, the inverse resistance becoming low; it is likely that failure was due to reverse breakdown.

Transient Response

In modulation systems the speed with which a switch will respond to an applied control signal is of great interest. The best way to measure this is to observe what happens to the transmitted r.f. when a square voltage pulse having a rapid rise and decay is applied to the switch. It is found that the response time varies with the magnitude of the applied pulse. Rise times (time taken to establish conduction in the diode) as low as 5 mμsec have been measured. The decay time is usually much longer, as it depends on the time taken for injected holes to diffuse out of the intrinsic region; it can be as long as 5 μsec. A better recovery time is obtained if a standing reverse bias is maintained; this helps to sweep out carriers. The recovery is further improved by limiting the forward-bias current, though in this case the rise time and maximum attenuation are degraded. The best figures obtained so far are rise and recovery times both of the order of 50 mμsec. For modulation of signals, the diode may not be driven into the non-linear region, and the response time is rather limited as a result. At present, satisfactory modulation at 100 kc/s has been achieved.

It might be thought that reducing the carrier lifetime would improve the transient response. Several diodes were made up with material which had been quenched to reduce carrier lifetime. However, the forward current at a given voltage was greatly reduced, and the degree of switching that could be attained was quite small. It seems that the greatest hope for improving switching speed lies in reducing the physical size.

Noise Performance

A diode in its waveguide unit has been tested to see if it would introduce noise into a receiving system. The diode did not noticeably impair the overall performance when under radar test conditions, nor was there any indication of flicker noise which would degenerate c.w. performance. Measurements were made both with zero bias and with −120-V bias on the diode.

Conclusions

The junction diode waveguide switch is a compact unit with modest control power requirements which will handle c.w. and pulsed microwave power. It should find many applications in radar and communication systems.

Acknowledgements

The author wishes to thank his colleagues for their assistance and encouragement.

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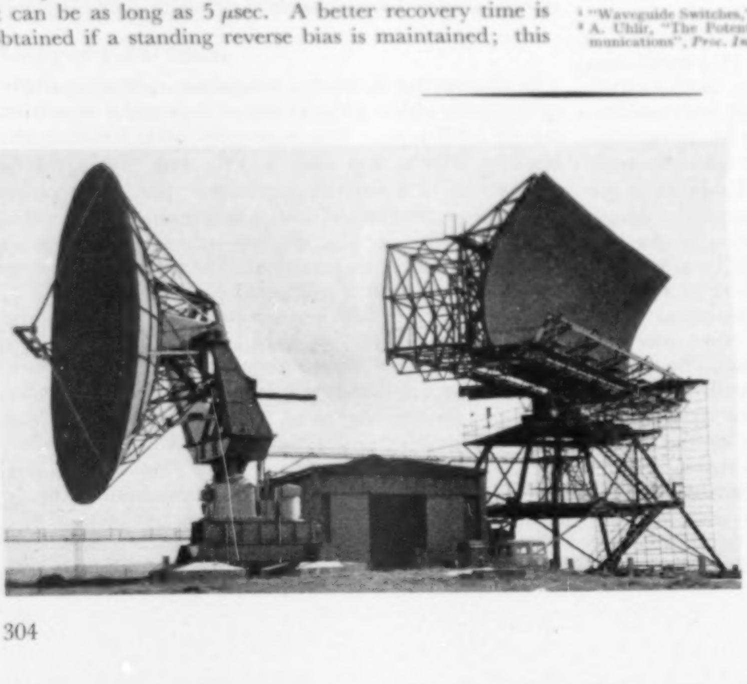

SATELLITE TRACKING EXPERIMENTS

As a preliminary in the study of communication by satellites, the Marconi Company is engaged in intensive studies of the ground equipment required in association with both active and passive satellites.

Shown here is a 30-ft. tracker dish and radar array at one of the Marconi research stations where radar tracking experiments are being conducted with passive satellites.

Correspondence

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Switching Transistors

SIR,—In connection with my article in your December 1960 issue, my attention has been drawn to some comments by Dr. J. R. Tillman (*Electronic Technology*, April 1961, p. 148) regarding the calculation of the charge associated with the collector depletion-layer capacitance when the collector–base voltage changes. The transistors discussed in my article were all of the type having abrupt junctions, and the theoretical treatment was therefore confined to this type. The relationship between collector depletion-layer capacitance and junction voltage was written in the form shown in equation (15) of my article in order to indicate clearly the dependence of this capacitance on the applied collector–base voltages, especially at low values of the latter. Apart from this small point I agree entirely with Dr. Tillman's remarks, and am pleased that he has drawn attention to the modifications needed in the calculations when dealing with junctions having linearly-graded impurity distribution, which would be a better representation in the case of transistors having collector junctions produced by diffusion methods. Usually, grown junctions would also fall into the latter category.

I consider that the significance of the charge contribution required by the junction capacitance is likely to become more and more important as the technology of switching transistors improves. Indeed, in the case of some of the most recent epitaxial switching transistors there is considerable evidence that such contributions may soon be the predominant ones.

J. N. BARRY.

The General Electric Company Limited,
Telecommunications and Engineering Laboratories,
Hirst Research Centre,
Wembley, England.
15th June 1961.

The Junction Transistor

SIR,—I have noted the comments which appeared in the February issue[1,2] on my article[3] and letter[4] on the operating mode of junction transistors as well as the Editorial in the same issue.

In the first part of my article[3], taking the common-emitter circuit as basic and others as derived versions, I have shown the common-emitter, common-base and common-collector circuits as the exact current-amplifier counterparts of the common-cathode, cathode-follower and common-grid valve circuits respectively.

In the second part of the article[3], it was shown that the maximum current amplification that can ever be obtained by applying direct positive feedback to a basic common-base amplifier will be restricted to $\alpha/(1-\alpha)$, and that a gain larger than this can be obtained only by using a current step-up device before feeding back the output to the input. On the other hand, applying positive current feedback to the common-emitter circuit whose feedbackless current amplification A $[A = \alpha/(1-\alpha)]$ is itself larger than unity, one can get any large current gain (even up to infinity; that is, when the circuit oscillates) without using a current step-up device between its output terminal (the collector) and the input terminal (the base), a feature not possible in the common-base connection. The example of the single transistor RC oscillator to illustrate this point and supplement the reasons for considering the common-emitter circuit as basic was perhaps not an elegant one, and it has caused a lot of confusion. It has led to digressions from the main theme to one of whether an RC circuit can step up current. (The brief but logical arguments in the second letter of Mr. Noordanus[1] pertaining to the current step-up property of a phase-inverting network are quite convincing and I can now appreciate his objections.) I would request you to bear with me and consider the following: A two-stage RC amplifier using junction transistors in identical common-emitter circuit arrangement would oscillate when the output is directly fed back to the input without any current step-up device (phase inverting and current step-up networks do not arise here). Can a similar two-stage amplifier employing common-base stages be made to oscillate without using a current step-up device in the feedback loop? And can the oscillations in the common-emitter arrangement be accounted for directly in terms of a basic common-base circuit?

The discussions below will further clear up matters: in junction transistor circuits, it is *only* the common-emitter circuit which has both current gain and voltage gain larger than unity (*all* amplifiers have power gain larger than unity) and it is *only* this circuit and *no other* which can be directly cascaded (without transformer coupling) for building multistage amplifiers to give increased current or voltage gain. In valve circuits the common-cathode circuit holds the same position. For this reason, the common-emitter circuit and the common-cathode circuit have *naturally* become the *basic building blocks* in junction transistor and valve amplifiers. Even if valve action had been first understood in terms of some other configuration, that configuration could not have become the basic building block in electronic amplifiers. In the case of the common-emitter circuit, it has inevitably become the basic unit in spite of junction-transistor action being first described in terms of some other configuration. Is it only a matter of opinion to consider these two circuits as basic in their respective families?

All the same, I would fully agree with you, Editor, that "the question of whether or not there is one mode of operation which can be regarded as the fundamental is not highly important". What mainly prompted me to write the article[3] and to urge the adoption of common-emitter operation as basic was the revelation of certain very fundamental relations in junction transistor circuits which this adoption would lead to and the help that the resulting clear thinking can give, at least to some circuit engineers, in circuit design and in the evolution of new circuits in a confusion-ridden field.

M. K. ACHUTHAN.

Dept. of Communication Engng,
Indian Institute of Technology,
Kharagpur, West Bengal, India.
11th April 1961.

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[2] R. R. Dutta Gupta, letter to *Electronic Technology*, February 1961, Vol. 38, p. 68.
[3] M. K. Achuthan, "The Junction Transistor", *Electronic Technology*, June 1960, Vol. 37, p. 238.
[4] M. K. Achuthan, letter to *Electronic Technology*, December 1960, Vol. 37, p. 474.

Heaviside's Method

SIR,—Shakespeare's remark, "What's in a name . . .?" (Romeo and Juliet II, ii) is very often quoted; less often remembered, perhaps, is the fact that the bard answered his own question: "Good name in man or woman, dear my lord, is the immediate jewel of their souls . . ." (Othello III, iii). For names are important: a good name, an apt name, a just name, is much to be desired for a man, for a thing, for a notion.

This train of thought about names was started by the reading of the recent very excellent articles by "Computer" in this Journal. These have occasion to use the techniques and notions first put to real use by Heaviside about 60 years ago. The whole body of techniques was at first known just as "Heaviside's method" or something like that. However, names such as "symbolic methods", "operational methods", and "Laplace Transformation" came to be applied. The last-named is especially common, nowadays, maybe because "transformation" is just now an "O.K. word" (to borrow an expression from Potter). How apt and just, however, are these various names?

It might be argued that "operational" or "symbolic" signifies very little; presumably the addition of two and two is an operation, and in doing it one could use symbols. What, on the other hand, is this Laplace transformation which of late ". . . doth bestride this narrow world" (of the analysis of transients) "like a Colossus . . ." ? The transformation of an expression is found by multiplying by a certain exponential expression, and integrating from zero to infinity. This will lead to an algebraic equation, which may be solved for the transformation of the unknown. Then the application of a certain process, the "inverse transformation" will give the desired solution.

Do people, then, ordinarily go through these steps when they want to solve a differential equation? Not in the least! They replace both sides of the equation by expressions in a new variable p according to certain rules, solve the resulting equation, and interpret the result according to other rules, or by referring to tables of results. The transformation, the multiplication by an exponential and integration from zero to infinity, never enters the question at all. It would seem, then, that to say that the problem has been solved by Laplace transforms is like saying that in multiplying two large numbers one was working with infinite series, on the grounds that the logarithms used were once, or could have been, computed with the help of infinite series.

What, it may be asked, is the point of these remarks? The methods ordinarily used in the analysis of transients, and similar problems, are those developed by Heaviside; changed, perhaps, slightly in notation, but still much the same. Should they not, then, be called after their inventor, and such methods and techniques go, as they did in the past, under the general name of "Heaviside's method", or words to that effect? Qui meruit palmam ferat.

H. L. Armstrong.

Department of Physics,
Queen's University, Kingston, Ontario, Canada.
24th April 1961.

Transistor Circuit Diagrams

Sir,—Those of us who teach the applied physicist and electronic engineers of the future must look forward rather than back. It is not easy to predict the nature of an advancing technology even five years ahead. Such evidence as exists, however, suggests that the student of today is much more likely during his professional career to be dealing with solid-state (and possibly electron-beam) devices than with conventional valves—at any rate so far as research and development is concerned. Consequently, it seems that the modern approach to teaching electronics should be to treat solid-state devices as the basic and 'normal' ones, and to relegate conventional thermionic valves to a subsidiary position in the course.

This change in policy has just been made in courses taught in this Department. First impressions are that the students find it no more difficult. Although the understanding of transistors requires a more extensive knowledge of physics than the understanding of valves, the discussion of transistors is not bedevilled by the need to rid the student's mind of preconceived ideas derived from valve circuits. It also seems possible to present transistor circuitry in such a way that the transition to valve circuitry is fairly smooth. The reverse process, in my experience, entails either much misunderstanding, or the presentation of valve circuits in an artificially complicated way.

The main difficulties, which have been encountered so far in carrying out this change, are a lack of textbooks on transistors which do not assume that the reader is familiar with at least the general principles of thermionic valves and valve circuits, and the effort required on the teacher's part in rethinking (and rewriting) his whole approach to the subject. However, at least one teacher can testify that the beneficial effects on his research activities made the effort worth while.

People, such as your correspondent Mr. T. G. Clark, who wish to avoid recognizing that this is a changing world, must realize that a 'vast accumulation of background knowledge' has to be either jettisoned or kept in the background every time a new advance is made, and that to say an 'emitter is a cathode' is neither more nor less illuminating than calling a motor car a horseless carriage. At best, both the direct and the dual analogy between transistors and valves give merely an indication of the correct line of attack on a problem; more often they are both quite misleading for reasons which I shall not attempt even to summarize here.

The question of sign conventions and their expression in circuit diagrams always raises heated arguments, but one thing is certain—any system must be as consistent as possible. If Mr. Clark wishes his rule to be 'collectors and anodes on top', good luck to him, especially when he has to deal with mixed n-p-n and p-n-p transistor circuits. I should be interested to know how his rule applies to diodes of the thermionic, p-n rectifying, tunnel, reverse junction, Zener or p-n-p-n switching variety. His solution to the problem (which does not exist if a reasonable convention is used) of showing pulse shapes in the circuit diagram is interesting. Does he also put in arrows to show the direction of current flow, and then add a suitable sign to indicate that the current really flows in the reverse direction?

But I must resist the temptation to be rude to Mr. Clark, in spite of his apparently extraordinary views on the interaction between physics and electronic engineering. Underlying his attitude there is a real difficulty which we all have to overcome—that of trying to keep abreast of new developments in a rapidly advancing technology. However, one sure way of falling behind is to attempt to perpetuate, not the unchanged basic principles, but outmoded rules of thumb.

J. C. S. Richards.

Department of Natural Philosophy,
University of Aberdeen.
17th June 1961.

New Books

Theory and Application of Ferrites

By Ronald F. Soohoo. Pp. 280 + vi. Prentice-Hall International Inc., 34-36 Beech Street, London, E.C.1. Price 63s.

The main idea expressed in the preface to Dr. Soohoo's book (i.e., to write a unified account of the engineering applications of ferrimagnetic materials in relation to their basic physical properties) is very good. It is, I am certain, an approach which will become increasingly common in 'solid-state engineering'.

Unfortunately, the treatment is not as a whole good. Clearly the author is very much at home with microwave behaviour and applications of ferrimagnetic materials—and it is the chapters devoted to these aspects which give the book its worth; by contrast, those devoted to more conventional magnetic behaviour are most disappointing.

The book is divided into two parts, roughly equal in length.

Following an Introduction, there are separate chapters devoted to Properties of matter in the solid state; Dielectric properties of ferrites; Magnetic properties of ferrites; and Eddy current, hysteresis, and residual losses in ferrites. The description in these chapters is adequate, but there is little evidence of a consistent attempt to integrate the various magnetic phenomena discussed.

Without doubt, the worst chapter of all in the book is that headed Measurements of ferrite properties. This gives a quite unbalanced and, in several cases, misleading description of measuring apparatus and techniques.

In one case, the author discusses the measurement of saturation magnetization by the "Sacksmith" balance (presumably, Sucksmith is meant). A reference is given to the 1939 paper in which Sucksmith first reported the use of the Ring Balance for measuring saturation moments, and it is inferred that the apparatus described by the author is identical. It is not.

On page 111, under the heading Measurement of Curie temperature, only one apparatus is described. The operation of this for measuring a Curie temperature is as follows:—A sample of the material is placed immediately below and in contact with a permanent magnet; below the Curie temperature it is held firmly by magnetic attraction. The assembly is gradually heated up, and the temperature at which the sample falls away from the magnet is called the Curie temperature. Use of this technique would be of little value for a scientific measurement of Curie temperature. In fact, the measurement of Curie temperatures is very difficult and requires refined techniques to obtain accurate values.

There is, too, some inconsistency. On page 92, a cavity method is described for measuring tensor permeabilities, and it is stated that the spherical specimen is "placed at the centre of the end plate . . ." of the cavity; yet, on page 97, in a more detailed description of this technique, we have "the ferrite sample . . . is inserted into the cavity spaced at about d or $2d$ from the back wall". These statements do not agree, but no mention is made in either description of the possibility of error in g-value determinations due to image effects changing the r.f. demagnetizing factor when the sample is in contact with the end wall of the cavity.

When one looks at that part of the book dealing with microwave propagation, devices and their applications, the picture is quite different, however, and the book comes alive.

Starting with a very good chapter on the Permeability tensor, we jump to Part 2 of the book on devices, applications and systems aspects. There are separate chapters on Longitudinal field devices; Transversely magnetized ferrite devices; Dielectric loading and tapered field devices; Cutoff phenomena in magnetized ferrites; Nonlinear ferrite devices and Systems application of ferrite devices. Of these, the chapters on Cutoff phenomena in magnetized ferrites and Systems applications are particularly valuable.

Several appendices contain derivations of various mathematical formulae quoted in the text.

Finally, I must criticize the style, and the quite large number of spelling mistakes—almost wholly in peoples' names. The style contains a very large amount of jargon. This is bad and, I think, reflects rather adversely on the editor of the series, of which this book forms part, as well as the author.

I would summarize my feelings by saying that the author's objects are wholly admirable, but he has not achieved them.

S.A.A.

Introduction to the Statistical Dynamics of Automatic Control Systems

By V. V. Solodovnikov. Translation edited by J. B. Thomas and L. A. Zadeh. Pp. 307 + xx. Constable & Co., 10-12 Orange Street, London, W.C.2. Price 18s.

The first 62 pages of this book review the mathematical theory of linear closed-loop systems, in terms of Fourier integral, Laplace transform, relationships between phase and amplitude characteristics and transfer function of a closed-loop. Chapter II gives a summary of probability theory, but unfortunately some confusions of terminology have arisen in the translation, and these could be dangerous to the student who is not well grounded in probability theory beforehand. For example, on p. 72 the definition of 'median' is very different from the normal usage of the term in Great Britain, and on p. 75 a quantity referred to as 'arithmetic mean' had in fact been correctly described earlier as the mean of the modulus of the quantity. Chapter III deals with stationary random processes and the correlation function, and it is here that one feels that progress has been made since the first publication of this book in Russian in 1952.

The definition of Markov processes is limited to those which would nowadays be called first-order Markov processes (i.e., future depending on the one immediate past value only); and apart from the rather inconvenient optical and mechanical correlators, the only instrument for measuring correlation which is described is the pulse-width and pulse-amplitude multiplier of Lee, Cheatham and Wiesner. Chapter IV gives a useful review of the response of systems to random inputs, including the effect of non-linearities such as the 'linear' detector and the square-law detector, and Chapter V discusses methods of approximating an experimentally determined function by a series of functions of convenient analytic form. Chapters VI, VII and VIII are concerned with the 'Wiener filter' problem; i.e., the synthesis of a transfer function to give minimum mean-square error for a given specification of signal and noise inputs.

The book contains a great deal of useful material, but it can hardly be regarded as up to date, since 9 years have elapsed since its original publication.

D.A.B.

Transistor Circuit Analysis

By Maurice V. Joyce and Kenneth K. Clarke. Pp. 461 + xiv. Addison-Wesley Publishing Co. Inc., 10–15 Chitty Street, London. W.1. Price 81s.

"This book is concerned with presenting the basic methods of analysis involved in the design and understanding of junction transistor circuitry. It is not a text in solid-state physics or transistor construction, nor is it just a compilation of a large number of transistor circuits. The transistor models employed in this analysis are restricted to a small number that are all easily interrelated. Initially, the simple low-frequency models are presented and are related to the underlying physical processes that occur in the device. Only after the reader is completely familiar with these simple models are the complications, such as high-frequency effects, noise, or saturation effects, introduced."

Principles of Transistor Circuits: *Introduction to the Design of Amplifiers, Receivers and other Circuits (2nd Edn)*

By S. W. Amos, B.Sc.(Hons.), A.M.I.E.E. Pp. 210 + x. Published for *Wireless World* by Iliffe Books Ltd., Dorset House, Stamford Street, London, S.E.1. Price 21s. (postage 1s. extra).

E.M.I. COMPUTER FILM

"Computer Achievements", a 22-minute film in colour with a sound commentary, has recently been produced by E.M.I. Electronics Ltd. Dealing broadly with the application of digital computers in commerce and industry, this film features five of the many varied uses for which the Emidec computers are being supplied to leading organizations. Copies of the film are available on loan to organizations with an English or Russian language sound commentary.

OBITUARIES

H. V. Griffiths, M.B.E., F.R.A.S., S.M.I.R.E., Engineer-in-Charge of the B.B.C. Measurement and Receiving Station at Tatsfield, died suddenly on 28th June at the age of 59.

Mr. Griffiths joined the B.B.C. in 1924 and was appointed Engineer-in-Charge at Tatsfield in 1933, where he remained throughout his career. As an authority on radio reception, propagation and frequency he represented the B.B.C. on a number of technical committees. In 1958 his work was recognized by the award of an M.B.E.

Philip Canning died on 1st July at the age of 59. He died at Basle Airport while returning from Interlaken where he had been presiding at a Technical Committee of the International Electrotechnical Commission.

Mr. Canning joined the Plessey Company in 1933 and from then until 1947 he was concerned with the development of aircraft radio apparatus and radio transmitters. Of recent years he has been responsible for co-ordinating Plessey activities and was Chairman of the Technical Committee of Radio Industry Exhibitions Ltd., Member of the Council (and past Chairman) of the R.E.C.M.F., Chairman of the Instruments, Electronics and Automation Exhibition Committee, as well as being Chairman of several standards committees.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Deviations from nominal frequency for June* 1961

Date 1961 June	MSF 60 kc/s 1500 U.T. Parts in 10^9	Droitwich 200 kc/s 1030 U.T. Parts in 10^9
1	− 15·3	+ 15
2	− 15·0	+ 15
3	− 15·0	+ 14
4	− 14·8	+ 14
5	− 15·1	+ 14
6	− 15·1	+ 14
7	− 15·1	+ 15
8	− 15·0	+ 15
9	− 14·9	+ 16
10	− 15·0	+ 16
11	− 15·1	+ 16
12	− 15·2	+ 15
13	− 15·0	+ 17
14	− 15·1	+ 16
15	− 15·3	+ 15
16	− 15·2	+ 16
17	− 15·2	+ 15
18	− 15·2	+ 15
19	− 15·1	+ 15
20	− 15·2	+ 16
21	− 15·2	+ 16
22	− 15·3	+ 15
23	− 15·2	+ 16
24	− 15·1	NM
25	− 14·9	NM
26	− 15·2	+ 13
27	− 15·3	+ 13
28	− 15·3	+ 13
29	− 15·3	+ 13
30	− 15·3	+ 13

* *Nominal frequency corresponds to a value of* 9 192 631 770 *c/s for the caesium F,m (4,0) — F,m (3,0) transition at zero field. NM — Not Measured.*

New Products

In-Situ Transistor Tester

The latest addition to the Advance range of instruments is the T.T.1 transistor tester which has been designed for checking low- and medium-power transistors while still in circuit.

Advance
b
c
e

With the T.T.1, both p-n-p and n-p-n transistors can be tested. Earthed-emitter current gain can be measured with the transistors in or out of circuit and leakage current (earthed-emitter) with the transistor out of circuit.

The tester provides for the measurement of current gain (β) in two ranges, 5 to 50 and 50 to 150 with an accuracy of ± 5% (with the transistor out of circuit). The set current range is 1 to 10 mA, but 10 mA is not obtainable with transistors of βs of below 12, and 1 mA is not obtainable where β exceeds 200. Leakage-current measurement ranges (earthed-emitter) are 0 to 1 mA and 0 to 100 μA, with the transistor out of circuit. The design of the tester is such that shunt-collector circuit resistances of greater than 100 Ω can be balanced out.

The T.T.L. measures 6 in. × 6½ in. × 7 in. and weighs 6 lb 6 oz.; it is battery-operated and provision is made for checking the condition of the batteries.

Advance Components Ltd.,
Roebuck Road, Ilford, Essex.

Water-Cooled R.F. Valves

Three r.f. power valves, with what is claimed to be a new more-efficient cooling method, have recently been added to the Ediswan range.

The valves are Type 15P12 with maximum anode dissipation of 0·6 kW, maximum r.f. power output of 2·5 kW and maximum operating frequency at full power of 60 Mc/s, and Types 16P12 and 16P13 both with maximum anode dissipation of 3 kW, maximum r.f.-power output of 7·5 kW, and maximum operating frequency at full power of 40 Mc/s. The 16P12 is connected to the water supply by flexible tubing and the 16P13 by unions. The maximum grid dissipation of the 15P12 is unusually high, since the grid is made of non-emitting material.

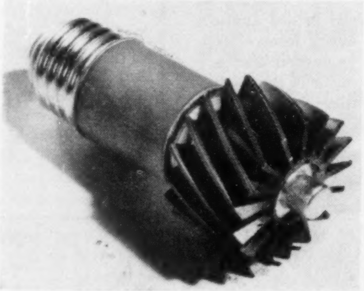
Instead of the conventional water jacket, these valves are constructed with a spiral copper water tube attached directly to the anode. The complete valve in each case is, therefore, more compact than comparable conventionally-cooled types and, because there are no seals, water-leakage risk is reduced.

Associated Electrical Industries Ltd.,
Radio & Electronic Components Division,
155 Charing Cross Road, London, W.C.2.

High-Voltage Ceramic Rectifiers

The latest developments in high-voltage vacuum ceramic envelope rectifiers from Ferranti include the UR. 41 rectifier and the DXV. 18432 experimental rectifier. Both of these valves have indirectly-heated cathodes, which are substantially metallic in nature, and combine features of conventional oxide-coated cathodes, which are economical in heating power, with those of thoriated-tungsten cathodes, which have the ability to withstand high voltages.

The UR. 41 was developed primarily for use as an inverse diode in radar modulators whenever conditions of extreme environments of high peak inverse voltages prevent the use of conventional glass valves. Rated at 16 kV p.i.v., the UR. 41 can withstand a surge voltage of up to 20 kV. The limiting anode temperature is 225 °C at which the valve has a life of 1,000 hr. The maximum peak pulse anode current rating is 14 A and the maximum anode dissipation is 40 W. The UR. 41 has an overall length of 105mm and a diameter over the cooling fins of 40 mm.



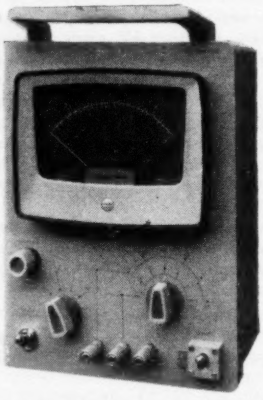
The DXV. 18432 is designed for use as an inverse diode in high-power radar modulators and is rated at 28 kV p.i.v. and 30 A peak pulse anode current. Air-cooled and liquid-cooled versions, with either Goliath Edison screw base or with flying leads, have been designed. The overall length is 7½ in. and without a cooler its maximum diameter is 1⅝ in. The maximum diameter when fitted with air cooling fins is 2½ in.

Ferranti Ltd.
Hollinwood, Lancs.

Frequency-Capacitance Meter

A portable battery-powered frequency/capacitance meter, recently produced by Solartron, gives direct readings of frequency (0–100 c/s) and capacitance (0–100 pF to 0–3 μF).

It is known as Type MM1009 and, as a frequency meter, it has a ± 3% measuring accuracy on 5 ranges up to 10 kc/s and ± 5% on the two ranges above. These quoted accuracies hold for 2 : 1 mark-space ratio. The input signal (100 mV–150 V r.m.s.) is limited and fed into a pulse-counting type integrator circuit.



As a capacitance meter, the MM1009 covers the range 0–0·3 μF in six ranges with an accuracy of ± 2·5% over the major part; below 300 pF, a 5-pF zero error must be taken into account. The unknown capacitor is compared with the internal standard by substitution in the integrator circuit.

The meter uses semiconductor devices throughout and is powered by eight U2 cells which give a useful operating life of about three months. Overall dimensions are

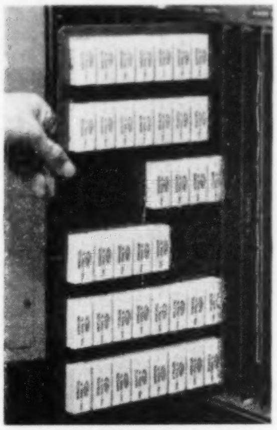
6¾ in. × 9⅞ in. × 5¼ in., and the weight is 6 lb.

Solartron Laboratory Instruments Ltd.,
Cox Lane, Chessington, Surrey.

Potted Transistorized Logic Elements

A fully-encapsulated version of the "Minilog" transistorized logic system for industrial sequencing and control applications has been introduced by Panellit.

Two types of Minilog elements are available. One has six inputs, giving an "inverted and" function, and drives up to five outputs; the other incorporates an emitter-follower and can drive up to twenty-five outputs. Both are exactly the same size (2 in. × 0·85 in. × 0·63 in.) and have eleven tinned-copper wire leads, 0·32 in. diameter. Transistor power elements can be supplied as top and bottom-end drivers for actuating solenoids and similar devices up to a maximum of 50 V at 1 A.



Forty-eight Minilog elements can be mounted on a board measuring 10 in. × 17 in. A special chassis, with gold-plated plugs and sockets to provide inter-board signal and power connections, can accommodate up to twelve of these boards.

Panellit Ltd.,
C/o Elliott Automation Ltd.,
34 Portland Place, London, W.1.

Solid-State Microwave Switches

A.E.I. have announced a range of silicon p-n junction-type microwave switches for use in such devices as TR cells. They employ variable-resistance diodes which enable them to handle high powers and they are operated from 1·5 V d.c.

The diodes will operate at any frequency from S- to X-band and are offered in tunable mounts to achieve maximum efficiency at any particular frequency in the band for which each mount is designed.

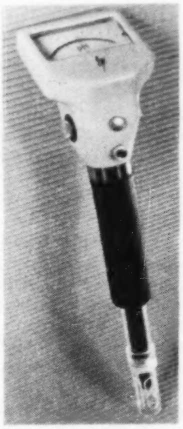
The properties of the diode in a waveguide mount are: typical insertion loss, 0·6–0·7 dB; attenuation, 26–30 dB; main waveguide power rating, pulsed 500 W, c.w. 10 W; diode rating 2 W; diode operating power 100 mW; switching time 0·5 μsec.

Associated Electrical Industries Ltd.,
Electronic Apparatus Division,
Crown House, Aldwych, London, W.C.2.

Portable pH Meter

A self-contained and truly portable pH meter of unusual design has recently been introduced into the U.K. by the H. G. Stevens Co. This is the Ludwig Seibold glass-electrode pH tester model GET.

The main departure in design lies in the construction—it is rather like a torch with a meter built into the top and the sensitizing electrodes attached to the bottom. This enables the instrument to be used in circumstances where speed and simplicity of operation are paramount.



The measuring range is 2 to 12 pH with a measuring accuracy of ± 0·1 pH (± 0·05 pH if specially calibrated). Equalization of temperature is provided and the control is hand-operated over the temperature range 5 to 80 °C.

Overall length is 18¼ in. and the weight, inclusive of batteries, is 3 lb 5 oz.

Distributed in the U.K. by

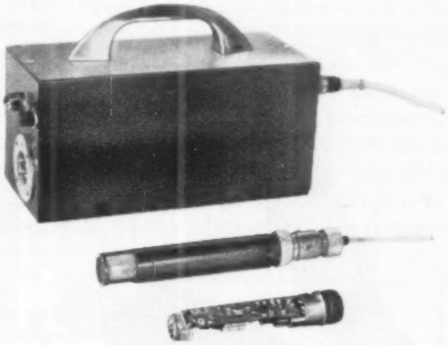
The H. G. Stevens Company Ltd.,
16 *Coverdale Road, London, N.W.2.*

S.T.C. Studio Microphones

Two studio-grade microphones have recently been announced by Standard Telephones & Cables.

The first, Model 4106, is a moving-coil unit with a cardioid polar response curve; the frequency range is from 30 c/s to 12 kc/s, the sensitivity is −85 dB relative to 1 V open circuit at 1 dyne/cm², the impedance is 30 Ω, and the back-to-front ratio is −20 dB. The approximate dimensions are 4·6 in. long by 1·6 in. diameter.

The second, known as 4108, is a condenser microphone with a cardioid polar response complete with an inbuilt transistor and valve head amplifier and a separate power unit. The frequency response is substantially level from 30 c/s to 15 kc/s and the sensitivity is −60 dB relative to 1 V open circuit at 1 dyne/cm²; the output impedance is 30 Ω or 300 Ω and the back-to-front ratio is −25 dB. The dimensions of the microphone and the combined head amplifier are 5·38 in. long by 0·88 in. diameter.

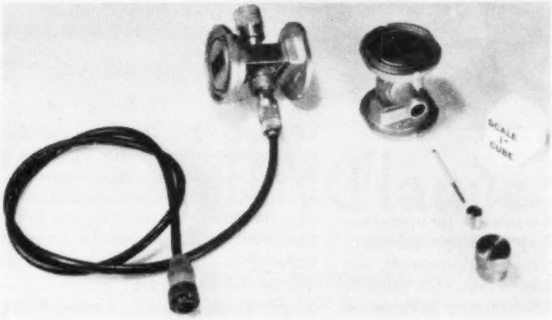


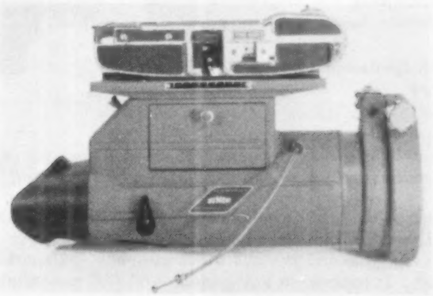
Standard Telephones & Cables Ltd.,
Aldwych, London, W.C.2.

Oscilloscope Cameras

Aveley Electric have recently announced the introduction of two DuMont oscilloscope cameras.

The first, Type 299, is for general-purpose recording from 5-in. tube oscilloscopes and





is available with a variety of backs for 120-roll or cut film.

The second is Type 302, also for 5-in. tube oscilloscopes, which incorporates a Polaroid-Land back for obtaining permanent records on transparent or opaque film in about a minute without a dark-room.

Each camera has a sliding back on the housing which permits the operator to adjust the camera in relation to the lens and expose more than one trace on the frame.

Detent stops position the camera back for equally-spaced exposures while a scale and pointer arrangement is provided for positioning the back accurately for more than three exposures per frame. Facilities for data recording are also provided.

The lenses available are the Woollensac DuMont 75-mm f/2·8 three-element lens or a 75-mm f/1·9 six-element lens.

Aveley Electric Ltd.,
Ayron Road, Aveley Industrial Estate,
South Ockendon, Essex.

Complex-Number Slide Rule

A slide rule which can be used to multiply or divide complex numbers with the same facility and accuracy as a conventional slide rule is used to multiply or divide real numbers is being marketed by Stanley & Co.

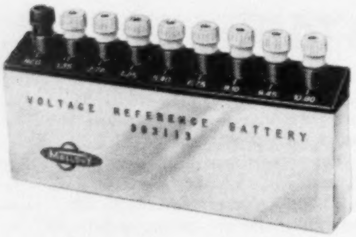
The slide rule comprises two cylinders, one fixed and the other movable, and two cursors. The numbers may be expressed in either rectangular or polar co-ordinates and the result is indicated in both rectangular and polar form. The maximum diameter is approximately 4 in. and the overall length (closed) is 17½ in.

W. F. Stanley & Co. Ltd.,
Avery Hill Road,
New Eltham, London, S.E.9.

Low-Impedance Voltage-Reference Battery

Designed for laboratory use as a stable voltage reference source, the Mallory mercury-type battery is claimed to be the first commercially-available low-impedance type.

Rugged, non-glass construction enables the battery to withstand brief short-circuits, severe vibration and physical shock without damage. It operates in any position and is fully portable, being supplied in a leatherette carrying case.



Two sizes are available—a single-cell 1·35-V battery and one with 8 outputs from 1·35 to 10·8 V in 1·35-V increments. An accuracy of ± ½% of stated open-circuit voltage and a temperature stability within 1% from 160 °F to −20 °F at currents up to 100 μA are claimed.

The Mallory reference battery is guaranteed for two years and will be supplied in the U.K. by Baird & Tatlock (London) Ltd., A. Gallenkamp & Co. Ltd., W. B. Nicolson (Scientific Instruments) Ltd. and J. W. Towers & Co. Ltd.

Mallory Batteries Ltd.,
Dagenham, Essex.

Transistorized Photoelectric Counter/Switch

A Series TPU.1 range of transistorized photoelectric equipment, which is available in alternative versions for counting or switching applications, has recently been announced by Lancashire Dynamo.

When used for counting applications, the unit is supplied complete with a Series F.43 electromagnetic counter, which is energized directly from a transistor switch circuit. In this configuration the unit is designed to operate up to a speed of 1,500 counts/min. Six-figure indication gives a total of 999,999 counts and manual reset facilities are provided.

In those applications where photoelectric switching is required, the unit is supplied with an integrally-mounted relay having one pair of normally open and one pair of normally closed contacts rated at 3 A, 230 V a.c. This unit is suitable for operation at switching rates of up to 300 per minute.

When used in conjunction with the standard 8-W light source unit, both versions are suitable for operation over a range of 25 ft; the light source unit obtains its supply directly from the photoelectric unit.

This equipment will operate from 100/115 or 200/250 V, 50 c/s mains supplies.

Lancashire Dynamo Electronic Products Ltd.,
Rugeley, Staffs.

30A Relay Contactor

A recent addition to the Keyswitch range of relays is a relay contactor which has been designed as a silent and robust unit for mains switching operations.

The switching contacts are rated at 30 A, 230 V and the unit measures 4in. × 2in. × 2in. It is fitted with screwed terminal connections and can be supplied enclosed in a transparent plastic cover.

KEYSWITCH RELAYS

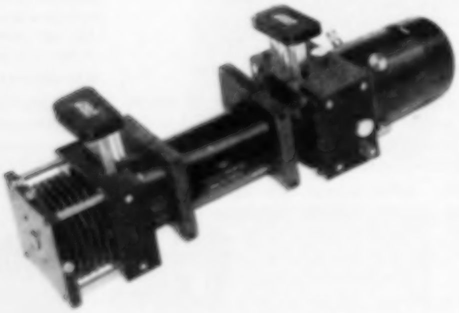
The Keyswitch Company,
2 Irongate Wharf Road,
London, W.2.

6-Mc/s Travelling-Wave Tube

The M-O Valve Co. have recently started production on an addition to their range of periodically-focused travelling-wave tubes.

It is known as the G.E.C. TWC5, a travelling-wave tube power amplifier, and is capable of delivering 15 W in the frequency band 5,925 to 6,425 Mc/s.

The TWC5 has been specially developed for multi-channel telecommunications systems operating in this range of carrier frequencies, although the permissible bandwidth coverage can be considerably greater. A main feature claimed for the tube is the ease with which it can be removed and replaced in a periodic magnetic circuit.



The M-O Valve Co. Ltd.,
Magnet House, Kingsway, London, W.C.2.

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With basic measuring instruments, such as the Slotted Line or Admittance Meter, generators, detectors, and a wide range of inter-related coaxial elements, all linked through the ingenious Type 874 connector, 'GENERAL RADIO' offer the scientist and engineer a 50-ohm VHF-UHF measuring system that is

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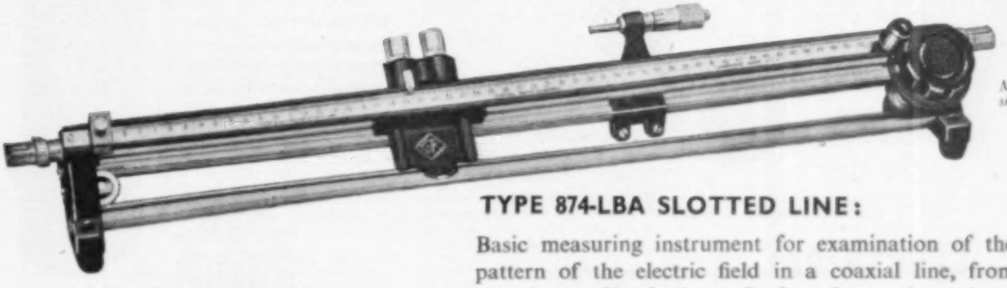
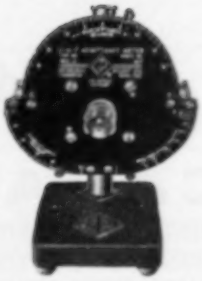


Micrometer Vernier shown attached.

TYPE 874-LBA SLOTTED LINE:

Basic measuring instrument for examination of the standing-wave-pattern of the electric field in a coaxial line, from which VSWR, impedance of load, phase of reflected wave, losses in attached elements, degree of mis-match between line and load, etc. can be determined. Accurate and straightforward in use. Frequency range 300-5000 Mc/s (with some loss in accuracy: 150-7000 Mc/s). Also available: Type 874-LV Micrometer Vernier (for measurement of high VSWR). Type 874-MD Motor Drive for oscillographic display of standing-wave pattern.



TYPE 1602-B ADMITTANCE METER:

A compact and versatile instrument, accurate and rapid in use, for determining the components of an unknown admittance in the VHF-UHF range. Scales read directly in conductance and susceptance, *independent of frequency*. With unknown connected through quarter-wave-length line, scales read in resistance and reactance. Can be used for measurement of VSWR and reflection coefficient, matching or comparison of impedances, and measurements on balanced line circuits (with the Type 874-UB 'Balun'). Frequency range 41-1500 Mc/s. (Down to 10 Mc/s, with correction). Includes conductance and susceptance standards

Write for complete information

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Keystone of the entire system is the unique Type 874 coaxial connector, fitted to all elements (see illustration below); this low-loss connector, any two of which, *although identical*, can be plugged together gives the system versatility and ease in setting-up for any measurement, and is characteristic of G-R's clear-sighted engineering philosophy. Low-loss adaptors are obtainable to link up with other systems.

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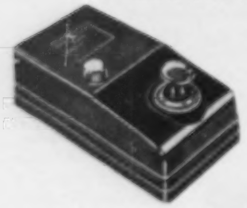

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Electronic technology

Press date for the September 1961 issue is first post 23rd August 1961

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CANDIDATES must hold City and Guilds certificates in Telecommunications or the O.N.C. (Electrical engineering), and must have had training and experience in electronic work, including, preferably, experience in high power transmission, broadcast practice or in the operation and maintenance of communications, transmitters and receivers and ancillary equipment; knowledge of telegraph printing machinery would be an advantage.

SELECTION will be by interview. The grade of appointment offered will depend upon qualifications and experience. All first appointments will be unestablished but there may be opportunities of establishment at a later date.

CANDIDATES and both their parents must have been British subjects or citizens of the Irish Republic at all times since birth.

CANDIDATES must be prepared to undergo a medical examination.

APPLICATIONS in writing, giving age, qualifications and experience should be addressed to the Personnel Officer, Diplomatic Wireless Service, Hanslope Park, Wolverton, Bucks. [1478

TUITION

BRADFORD INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering

A full-time post-graduate course, of twelve weeks' duration, on Transistors and Transistor Circuit Design will be held commencing 18th September 1961. Further information may be obtained from the Head of the Department of Electrical Engineering, Bradford Institute of Technology, Bradford, 7. [1479

BATTERSEA COLLEGE OF TECHNOLOGY LONDON, S.W.11
(Spectroscopy Section)

A course of 6 evening lectures on "Solid State Devices" will be given on Thursdays, 7 p.m., commencing 5th October, 1961. Fee 10s. Details and application forms may be obtained from the Secretary (Spectroscopy Courses). [1481

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AN Electronic Digital Computer constructed for the Imperial College of Science & Technology is offered for sale. Specification: 4,000 word drum, 32 bits per word; one address code plus one; optimum programming. Suitable for teaching elements of programming and for illustrating circuit principles of computers. Inquiries to the Financial Secretary, Imperial College of Science & Technology, London, S.W.7. [1480

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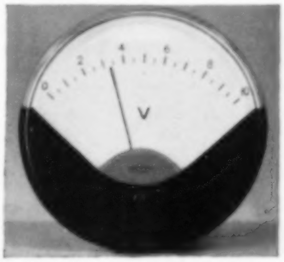

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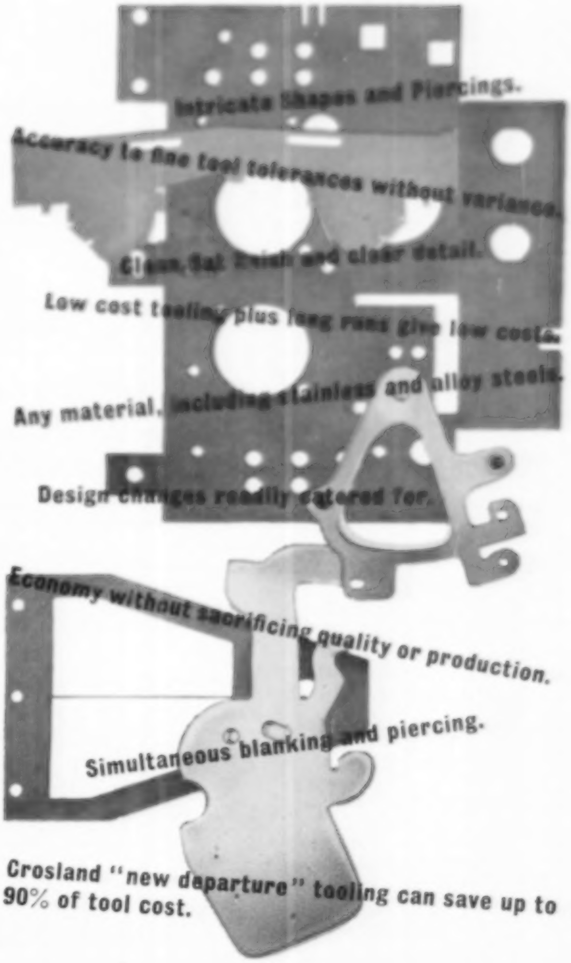

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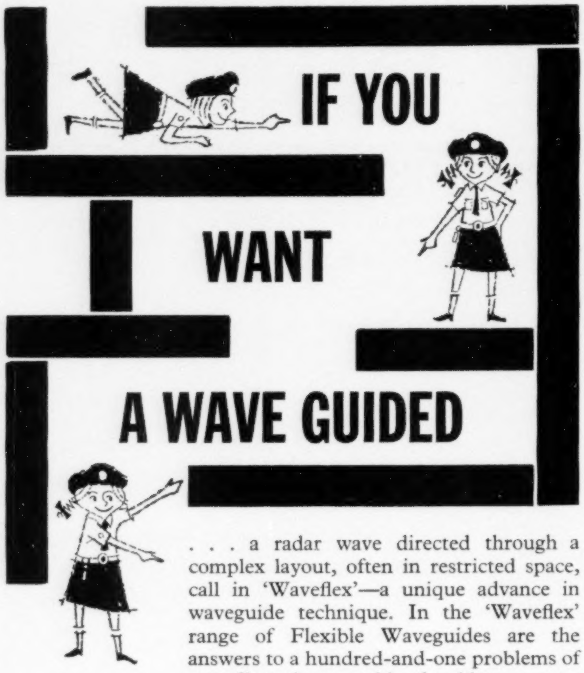

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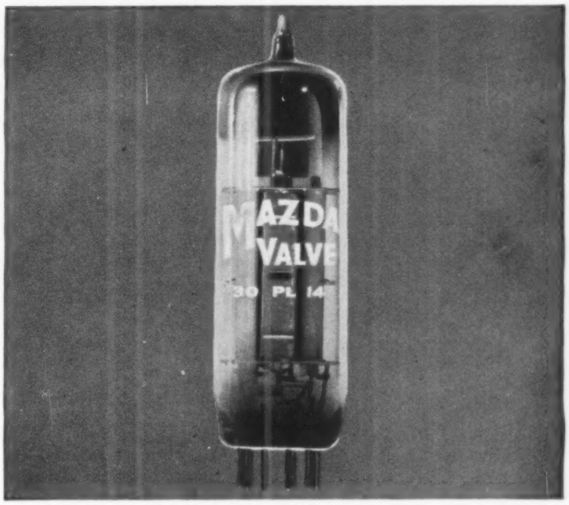

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Heater Current (amps)	I_h	0.3
Heater Voltage (volts)	V_h	16

TENTATIVE RATINGS AND DATA

Maximum Design Centre Ratings

		Triode	Tetrode
Anode Dissipation (watts)	$p_{a(max)}$	1	8
Screen Dissipation (watts)	$p_{g2(max)}$	—	2
Peak Anode Voltage (Pulse positive) (kV)		—	2*
Peak Anode Voltage (Pulse negative) (kV)		—	0.5*
Anode Voltage (volts)	$V_{a(max)}$	250	250
Screen Voltage (volts)	$V_{g2(max)}$	—	250
Heater to Cathode Voltage (volts rms)	$V_{h\text{-}k(max)rms}$	150†	150†
Mean Cathode Current (mA)	$I_{k(av)max}$	—	75
Resistance Grid 1 to Cathode			
Self Bias (MΩ)		—	2
Fixed Bias (MΩ)		—	1

* Maximum pulse duration 5% of one cycle with a maximum of 1 msec.

† Measured with respect to the higher potential heater pin.

TRIODE CHARACTERISTICS

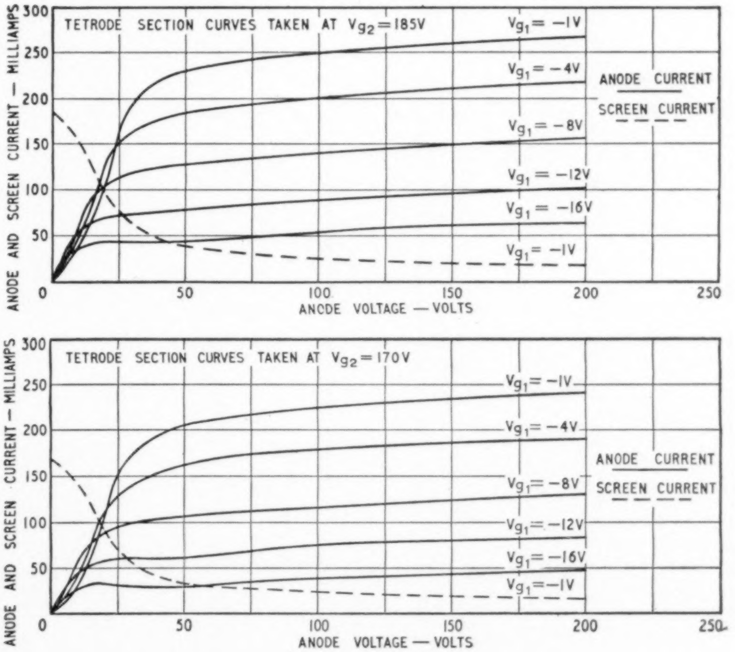
Anode Voltage (volts)	V_a	200
Anode Current (mA)	I_a	10
Grid Voltage (volts)	V_{g1}	7.7
Mutual Conductance (mA/V)	g_m	3.4
Amplification Factor	μ	18

TETRODE OPERATION IN FRAME TIME BASE

Allowance must be made in circuit design, not only for component variation, but for valve spread and deterioration during life. Values of total tetrode peak anode current, for an average valve when new and at the assumed end of life point for any valve, are as follows :—

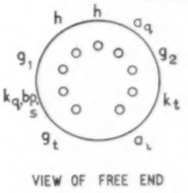
	V_a (V)	V_{g2} (V)	V_{g1} (V)	I_a (mA)
Average New Valve	55	170	− 1	210
Assumed End of Life Condition	50	170	− 1	135
Average New Valve	55	185	− 1	235
Assumed End of Life Condition	50	185	− 1	151

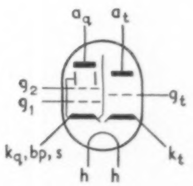
Tentative Characteristic Curves of Ediswan Mazda Valve Type 30PL14



Mounting Position: Unrestricted

Base: B9A (Noval)





Maximum Dimensions (mm)

Overall Length	78.5
Seated Height	71.5
Diameter	22.2

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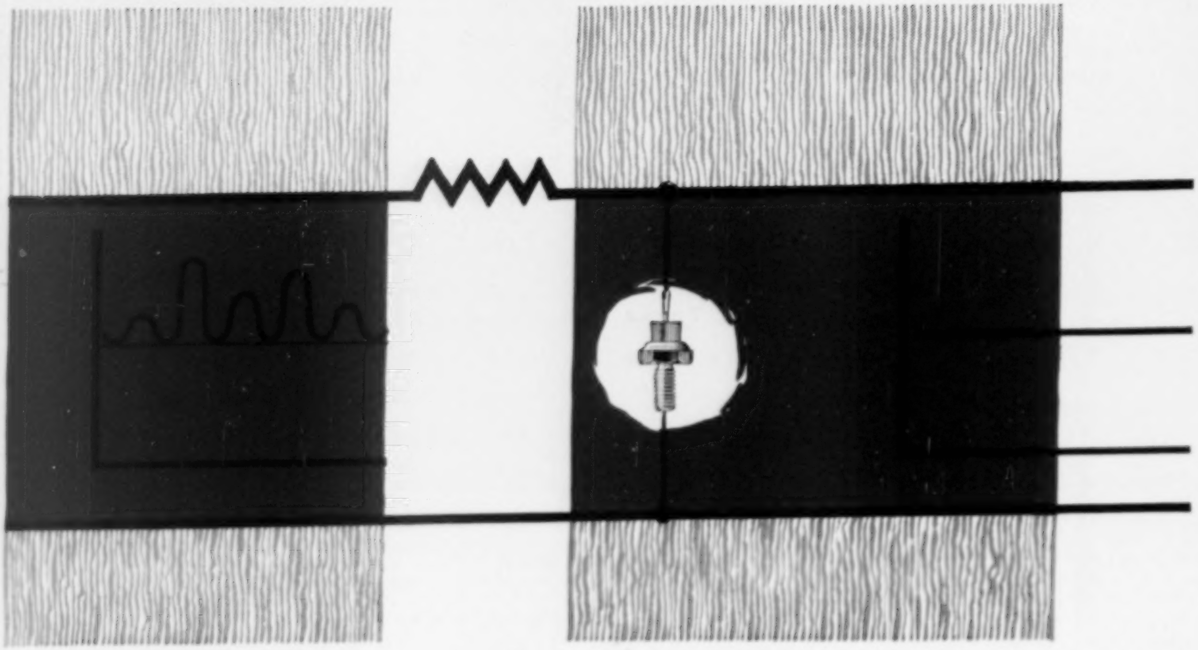

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ZD 510 CD510	10	10
ZD 512 CD 512	12	10
ZD 518 CD 518	18	10
ZD 522 CD 522	22	10
ZD 527 CD 527	27	10
ZD 533 CD 533	33	10

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